ERRATA TO:

***STRAPDOWN ANALYTICS*, FIRST EDITION**

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PARTS 1 AND 2

STRAPDOWN ASSOCIATES, INC. 2000

Corrections Reported From April 2004 To Present

Pg. 10-45 - Section 10.1.3, first paragraph - Frequency folding is when the oscillation frequency is a multiple of the sample frequency, not the inverse as in the book.

Sections 12.3.6.1 and 12.3.6.2 - The section titles are interchanged. The Section 12.3.6.1 title should be "E FRAME DEFINED ERROR PARAMETER RATE EQUATIONS FROM N FRAME DEFINED PARAMETER RATE EQUATIONS". The Section 12.3.6.2 title should be "N FRAME DEFINED ERROR PARAMETER RATE EQUATIONS FROM E FRAME DEFINED PARAMETER RATE EQUATIONS".

Pg. 18-33 - The equation in the last sentence should be (18.2.1-25).

Corrections Reported From January 2004 To April 2004

Pg. 18-102 - Delete the minus sign in front of $\frac{1}{2\,g}$ in the μ_{zy} equation.

Corrections Reported From November 2003 To January 2004

Pg. B-2 (Volumes 1 and 2) - Add following Attitude: "Average of Averages filter. 18.4.7.3"

Pg. B-7 (Volumes 1 and 2) - Add to Quantization error (on inertial sensor outputs), Defined: "18.4.7.3"

Corrections Reported From June 2003 To November 2003

Pg. 18-84, Table 18.4.1-2 - Axis Along Outer Rotation Fixture Axis for SEQUENCE NUMBER 20 should be -Y.

Pg. 16-15 - Equation (16.1.1.4-2) should be:

Conditions For Zero x Coupling Into Δx, Δy:

$$L_x = I_x \qquad H_x^* = H_x$$

or

$$\Phi_{xx}^* = \Phi_{xx} \qquad \Phi_{yx} = 0 \qquad H_x^* = H_x \tag{16.1.1.4-2}$$

or

$$L_x = \text{Diagonal} \qquad H_x^* = H_x$$

$$\text{If } \left(L_x(i{=}j, j) \neq 1\right) \text{ Then } \Phi_{xx}^*(i,j) = \Phi_{xx}(i,j) \text{ And } \Phi_{yx}(i,j) = 0 \text{ For All i}$$

Corrections Reported From 2000 To June 2003

Pg. 3-44, last line - Should be: "From Equation (3.2.4-13), the conclusion following (3.2.4-20), and the above definitions we have:"

Pgs. 7-33, 7-34, 7-58, 7-59, 7-60, and 17-83 - Correction To The Navigation Frame Rotation Compensation Terms In The Velocity And Position Integration Algorithms

The following is a revision to pages dealing with the small correction applied during acceleration transformation operations to account for rotation of the L Frame during the m cycle.

The term of concern is $\left(C_{L_{I(n-1)}}^{L_{I(m)}} - I\right) \Delta \underline{v}_{SF_m}^{L_{I(n-1)}}$ in Equation (7.2.2-4) which is incorrect in the form shown. The correct term is developed by rederiving Equation (7.2.2-1). Using the matrix chain rule and integration by parts:

$$\Delta \underline{v}^{L}_{SF_m} = \Delta \underline{v}^{L_{I(m)}}_{SF_m} = \int_{t_{m-1}}^{t_m} C^{L(t)}_{L_{I(m-1)}} C^{L_{I(m-1)}}_{L_{I(n-1)}} C^{L_{I(n-1)}}_{B_{I(m-1)}} C^{B_{I(m-1)}}_{B(t)} \underline{a}^{B}_{SF} \, dt$$

$$= C^{L_{I(m)}}_{L_{I(m-1)}} C^{L_{I(m-1)}}_{L_{I(n-1)}} C^{L_{I(n-1)}}_{B_{I(m-1)}} \int_{t_{m-1}}^{t_m} C^{B_{I(m-1)}}_{B(t)} \underline{a}^{B}_{SF} \, dt \qquad (7.2.2\text{-}1)$$

$$- \int_{t_{m-1}}^{t_m} \dot{C}^{L(t)}_{L_{I(m-1)}} C^{L_{I(m-1)}}_{L_{I(n-1)}} C^{L_{I(n-1)}}_{B_{I(m-1)}} \int_{t_{m-1}}^{t} C^{B_{I(m-1)}}_{B(\tau)} \underline{a}^{B}_{SF} \, d\tau \, dt$$

Assuming $\dot{C}^{L(t)}_{L_{I(m-1)}}$ is approximately constant and $\int_{t_{m-1}}^{t} C^{B_{I(m-1)}}_{B(\tau)} \underline{a}^{B}_{SF} \, d\tau$ changes approximately linearly over the m cycle we then get:

$$\Delta \underline{v}^{L}_{SF_m} \approx C^{L_{I(m)}}_{L_{I(m-1)}} C^{L_{I(m-1)}}_{L_{I(n-1)}} C^{L_{I(n-1)}}_{B_{I(m-1)}} \int_{t_{m-1}}^{t_m} C^{B_{I(m-1)}}_{B(t)} \underline{a}^{B}_{SF} \, dt$$

$$- \int_{t_{m-1}}^{t_m} \frac{1}{T_m} \left(C^{L_{I(m)}}_{L_{I(m-1)}} - I \right) C^{L_{I(m-1)}}_{L_{I(n-1)}} C^{L_{I(n-1)}}_{B_{I(m-1)}} \left(\int_{t_{m-1}}^{t_m} C^{B_{I(m-1)}}_{B(\tau)} \underline{a}^{B}_{SF} \, d\tau \right) \frac{(t - t_{m-1})}{T_m} \, dt$$

$$= C^{L_{I(m)}}_{L_{I(m-1)}} C^{L_{I(m-1)}}_{L_{I(n-1)}} C^{L_{I(n-1)}}_{B_{I(m-1)}} \int_{t_{m-1}}^{t_m} C^{B_{I(m-1)}}_{B(t)} \underline{a}^{B}_{SF} \, dt$$

$$- \frac{1}{2} \left(C^{L_{I(m)}}_{L_{I(m-1)}} - I \right) C^{L_{I(m-1)}}_{L_{I(n-1)}} C^{L_{I(n-1)}}_{B_{I(m-1)}} \int_{t_{m-1}}^{t_m} C^{B_{I(m-1)}}_{B(t)} \underline{a}^{B}_{SF} \, dt \qquad (7.2.2\text{-}1a)$$

$$= \frac{1}{2} \left(C^{L_{I(m)}}_{L_{I(m-1)}} + I \right) C^{L_{I(m-1)}}_{L_{I(n-1)}} C^{L_{I(n-1)}}_{B_{I(m-1)}} \int_{t_{m-1}}^{t_m} C^{B_{I(m-1)}}_{B(t)} \underline{a}^{B}_{SF} \, dt$$

$$= \frac{1}{2} \left(C^{L_{I(m)}}_{L_{I(n-1)}} + C^{L_{I(m-1)}}_{L_{I(n-1)}} \right) C^{L_{I(n-1)}}_{B_{I(m-1)}} \int_{t_{m-1}}^{t_m} C^{B_{I(m-1)}}_{B(t)} \underline{a}^{B}_{SF} \, dt$$

With this result, Equations (7.2.2-2) and (7.2.2-3) are unchanged but Equation (7.2.2-4) becomes:

$$\Delta \underline{v}^{L}_{SF_m} = \frac{1}{2} \left(C^{L_{I(m)}}_{L_{I(n-1)}} + C^{L_{I(m-1)}}_{L_{I(n-1)}} \right) \Delta \underline{v}^{L_{I(n-1)}}_{SF_m}$$

$$= \Delta \underline{v}^{L_{I(n-1)}}_{SF_m} + \frac{1}{2} \left[\left(C^{L_{I(m)}}_{L_{I(n-1)}} - I \right) + \left(C^{L_{I(m-1)}}_{L_{I(n-1)}} - I \right) \right] \Delta \underline{v}^{L_{I(n-1)}}_{SF_m} \qquad (7.2.2\text{-}4)$$

In the fifth line of Section 7.2.2.1, the analytical term $\left(C^{L_{I(m)}}_{L_{I(n-1)}} - I\right) \Delta\underline{v}^{L_{I(n-1)}}_{SF_m}$ should then be:

$$\frac{1}{2}\left[\left(C^{L_{I(m)}}_{L_{I(n-1)}} - I\right) + \left(C^{L_{I(m-1)}}_{L_{I(n-1)}} - I\right)\right] \Delta\underline{v}^{L_{I(n-1)}}_{SF_m}$$

Also change $\left(C^{L_{I(m)}}_{L_{I(n-1)}} - I\right)$ in the sixth line of Section 7.2.2.1 to the previous expression.

For the position update in Section 7.3.3, the $\Delta\underline{v}^{L}_{SF}(t)$ equation in (7.3.3-3) should be:

$$\begin{aligned}
\Delta\underline{v}^{L}_{SF}(t) &= \frac{1}{2}\left(C^{L(t)}_{L_{I(n-1)}} + C^{L(m-1)}_{L(n-1)}\right) \Delta\underline{v}^{L(n-1)}_{SF}(t) \\
&= \frac{1}{2}\left(C^{L(t)}_{L(m-1)} + I\right) C^{L(m-1)}_{L(n-1)} \Delta\underline{v}^{L(n-1)}_{SF}(t) \\
&= \frac{1}{2}\left(C^{L(t)}_{L(m-1)} - I\right) C^{L(m-1)}_{L(n-1)} \Delta\underline{v}^{L(n-1)}_{SF}(t) + C^{L(m-1)}_{L(n-1)} \Delta\underline{v}^{L(n-1)}_{SF}(t) \\
&= \frac{1}{2}\left(C^{L(m)}_{L(m-1)} - I\right) \frac{\left(t - t_{m-1}\right)}{T_m} C^{L(m-1)}_{L(n-1)} \Delta\underline{v}^{L(n-1)}_{SF_m} \frac{\left(t - t_{m-1}\right)}{T_m} + C^{L(m-1)}_{L(n-1)} \Delta\underline{v}^{L(n-1)}_{SF}(t) \\
&= \frac{1}{2}\left(C^{L(m)}_{L(m-1)} - I\right) C^{L(m-1)}_{L(n-1)} \Delta\underline{v}^{L(n-1)}_{SF_m} \frac{\left(t - t_{m-1}\right)^2}{T_m^2} + C^{L(m-1)}_{L(n-1)} \Delta\underline{v}^{L(n-1)}_{SF}(t) \\
&= \frac{1}{2}\left(C^{L(m)}_{L(n-1)} - C^{L(m-1)}_{L(n-1)}\right) \Delta\underline{v}^{L(n-1)}_{SF_m} \frac{\left(t - t_{m-1}\right)^2}{T_m^2} + C^{L(m-1)}_{L(n-1)} \Delta\underline{v}^{L(n-1)}_{SF}(t)
\end{aligned}$$

and the $\Delta\underline{R}^{L}_{SF_m}$ equation in (7.3.3-4) becomes:

$$\Delta\underline{R}^{L}_{SF_m} = -\frac{1}{6}\left[\left(\underline{\zeta}_{n-1,m} - \underline{\zeta}_{n-1,m-1}\right)\times\right] \Delta\underline{v}^{L(n-1)}_{SF_m} T_m + C^{L(m-1)}_{L(n-1)} C^{L(n-1)}_{B(m-1)} \Delta\underline{R}^{B}_{SF_m}$$

The effect on the Chapter 17 trajectory generator is:

$$\mathfrak{C}^{L_m}_{L_{m-1}} \approx I - \frac{1}{2}\left(\underline{\zeta}_{v_m}\times\right) \qquad (17.2.3.1\text{-}10)$$

$$\Delta\underline{R}^{L}_{SF_m} = -\frac{1}{6}\underline{\zeta}_{v_m} \times \Delta\underline{v}^{L_{m-1}}_{SF_m} T_m + \left(C^{L}_{B}\right)_{m-1} \Delta\underline{R}^{B}_{SF_m} \qquad (17.2.3.1\text{-}17)$$

The previous corrections should also be applied to the equivalent terms in Equations (17.2.3.1-30) and (17.2.3.1-31).

Pg. 8-6, last line - Should be replaced by: "$\delta\underline{\omega}_{Bias}$ is determined by first writing from (8.1.1.1.1-8):"

Pg. 8-7 - In the first sentence add "then" before "yields". The line preceding Equation (8.1.1.1.1-12) should read: "Applying (8.1.1.1.1-11) in (8.1.1.1-7) also obtains for $\underline{K}_{Bias}$ in alternate compensation Equations (8.1.1.1-8):"

Pg. 8-8, paragraph following Equation (8.1.1.1.1-16) - Add at the end: "Equations (8.1.1.1-5) and (8.1.1.1-6) can then be used to obtain Ω_{Wt} and K_{Mis} in alternate compensation Equations (8.1.1.1-8)."

Pg. 8-30, first definition following Equation (8.1.3.1-4) - Delete: "(or pulse count)".

Pg. 10-150 - Include the F(Ω) equation from (10.6.2-20) in the (10.6.2-21) summary equation set.

Pg. 11-14, Equations (11.2.1.1-5) - The following should replace the equivalent terms in the original version:

$$I\omega_{YR} = \omega_c \sin\beta \frac{1}{\dot{\phi}} \left(\cos\phi - \cos\alpha\right) \qquad I\omega_{ZR} = -\,\omega_c \sin\beta \frac{1}{\dot{\phi}} \left(\sin\phi - \sin\alpha\right)$$

Pg. 11-46 - Add to the end of the first sentence following the Equation (11.2.4.3.1.1-1) definitions: "· · · premultiplied by C_L^N."

Pg. 11-48, definition at top of page - Delete: "(positive in the plus ϕ direction)".

Pgs. 13-24 and 13-25 - Change (13.2.4-16) to:

$$\underline{\psi}^{I}_{TMis} = \left(C^I_B - C^I_{B_0}\right) \delta\underline{K}_{TMis} \qquad (13.2.4\text{-}16)$$

Delete the $\Delta\underline{\psi}^{I}_{TMis}$ definition following (13.2.4-16) and add the following in place of the last sentence beginning on page 13-24:

"The same effect holds for any coordinate frame in which $\underline{\psi}$ is being evaluated, not only the I Frame, provided that rotation of the frame is properly taken into account. For example, for the local level L Frame, Equation (13.2.4-16) becomes:

$$\begin{aligned} \underline{\psi}^{L}_{TMis} &= C^L_I \left(C^I_B - C^I_{B_0}\right) \delta\underline{K}_{TMis} = \left(C^L_B - C^{LI}_{B_{I0}}\right) \delta\underline{K}_{TMis} \\ &= \left(C^L_B - C^{LI}_{L_{I0}} C^{L_{I0}}_{L_I} C^{L_I}_{B_{I0}}\right) \delta\underline{K}_{TMis} = \left(C^L_B - C^{LI}_{L_{I0}} C^{L_{I0}}_{B_{I0}}\right) \delta\underline{K}_{TMis} \\ &= \left(C^L_B - C^{LI}_{L_{I0}} C^{L_0}_{B_0}\right) \delta\underline{K}_{TMis} \end{aligned} \qquad (13.2.4\text{-}16a)$$

where

L_0 = L Frame orientation in inertial space at time t = 0.

and for more specificity,

L_I = Discrete attitude of the L Frame in non-rotating inertial space at the current time.

B_{I0} , L_{I0} = Discrete attitudes of the B and L Frames in non-rotating inertial space at time t = 0.

The previous notation is introduced to clearly denote $C_{L_{I0}}^{L_I}$ as being a relative attitude matrix between two time points evaluated in the I Frame. Without the I notation (e.g., $C_{L_0}^{L}$) the matrix would have different values depending on the coordinate frame in which it was evaluated. For example, contrast Equations (7.1.1.2-3) and (7.1.1.2-4) with (7.3.1-8) and (7.3.1-9) (substituting L for N) while recognizing that $\underline{\rho} \equiv \underline{\omega}_{EN} = \underline{\omega}_{EL}$."

Pg. 14-80, Paragraph following table - Should read: "We see from the table that the maximum for the error function occurs near $q_2 / q_3 = 3$ for which the error is - 10.9% (i.e., the approximate sum solution $P_{\Omega_{Sum}}$ is 10.9% smaller than the true solution $P_{\Omega_{Simult}}$). The standard deviation of the Ω_{Sum} error at $q_2 / q_3 = 3$ (i.e., the square root ratio $\sqrt{\frac{P_{\Omega_{Sum}}}{P_{\Omega_{Simult}}}}$ compared to one) is - 5.6%. For large or small values of q_2 / q_3 the error in the $P_{\Omega_{Sum}}$ solution is zero. Thus, we see that use of $P_{\Omega_{Sum}}$ as an approximation to $P_{\Omega_{Simult}}$ has little error over the full range of q_2 / q_3."

Pg. 15-118 - Delete the last paragraph.

Pg. 15-131 - Add the following at the beginning of the first sentence following Equation (15.2.2.2-4): "Under horizontal maneuvering, · · ·"

Pg. 15-133 - Add the following at the end of the second paragraph: "Also note that by continuously updating C_N^E during the alignment process, $\underline{\varepsilon}^N$ is implicitly controlled to zero, thereby validating the $\underline{\varepsilon}^N = 0$ approximation of the previous paragraph."

Pg. 16-33 to 16-35 - The first equation in (16.2.3.1-6) and Equation (16.2.3.1-8) should be:

$$\dot{\underline{\psi}}^N = \cdots + \frac{1}{2} C_B^N \left[\underline{\omega}_{IB}^B \times \left(\delta\underline{\alpha}_{\psi Quant} + \delta\underline{\alpha}_{\psi VQuant} \right) \right] + \cdots$$

$$G_{P_{\psi V}} = \begin{bmatrix} \frac{1}{2} C_B^N \left(\underline{\omega}_{IB}^B \times \right) & \frac{1}{2} C_B^N \left(\underline{\omega}_{IB}^B \times \right) \\ 0 & - \left(\underline{a}_{SF}^N \times \right) C_B^N \end{bmatrix} \qquad (16.2.3.1\text{-}8)$$

The 1/2 factor in the previous expressions is the ratio between the velocity and attitude update frequencies and is needed because there is 1 contribution of $\delta\underline{\alpha}_{\psi\mathrm{Quant}}$ and 1 contribution of $\delta\underline{\alpha}_{\psi\mathrm{VQuant}}$ (i.e., 1 + 1 = 2 quantization noise contributions) to the attitude update cycle for each velocity update cycle. Equation (16.2.3.1-9) provided in the book is consistent with this change. The same effect also applies to Equation (16.2.3.1-11) which becomes:

$$G_{P_{VR}} = \begin{bmatrix} -\frac{1}{r}\left[\left(C_B^N\,\underline{\omega}_{IB}^B\right)\times\right]C_B^N & -\frac{1}{r}\left[\left(C_B^N\,\underline{\omega}_{IB}^B\right)\times\right]C_B^N \\ 0 & C_B^N \end{bmatrix} \qquad (16.2.3.1\text{-}11)$$

The 1/r multiplier in (16.2.3.1-11) arises because there are r-1 contributions of $\delta\underline{\upsilon}_{\mathrm{VQuant}}$ and 1 contribution of $\delta\underline{\upsilon}_{\mathrm{VRQuant}}$ (i.e., r-1 + 1 = r quantization noise contributions) to the velocity update cycle for each position update cycle. Equation (16.2.3.1-12) provided in the book is consistent with the previous change.

Move the r definition following Equation (16.2.3.1-12) up to the definition group following Equation (16.2.3.1-11).

Pg. 17-3 - Delete the fourth bullet line.

Pg. 17-17 - At the end of the second sentence following the top of page definition, change "divided by the velocity." to "divided by the great circle angular velocity. The angular velocity equals the velocity along the great circle divided by the great circle radius." Change Equation (17.1.1.4-6) to:

$$T_{S/GC} = \frac{R_{Avg}\,\theta_{GC/Range}}{V_{GC}} \qquad (17.1.1.4\text{-}6)$$

Add the following definition at the bottom of the page:

R_{Avg} = Average distance to earth's center over the $\theta_{GC/Range}$ angle (See Table 5.6-1 R calculation). Can be approximated as R at the start of the trajectory segment.

Pg. 17-18 - In the second sentence in the second paragraph, change "velocity" to "velocity magnitude".

Pg. 17-19 - Add the following at the start of the first sentence: "Recognizing that during the turn $\underline{u}_{X\mathcal{V}}^{\mathcal{V}}$ is horizontal, perpendicular to the vertical $\underline{u}_{ZL}^{\mathcal{V}}$, hence $\left(\underline{u}_{ZL}^{\mathcal{V}} \times \underline{u}_{X\mathcal{V}}^{\mathcal{V}}\right)$ is a unit vector,"

Pg. 18-20 - Add a closing bracket $\big\}$ to the first equation in (18.1.2-23).

Pg. B-1 (Volumes 1 and 2) - Add to Accelerometer error characteristics: "12.4"

Pg. B-2 (Volumes 1 and 2) - Add to Angular rate sensor error characteristics: "12.4"

Pg. B-4 (Volumes 1 and 2) - Under Free azimuth coordinates (defined), change "2.2" to "4.5"

Pg. B-7 (Volumes 1 and 2) - Add to Schuler, Dr. Maximilian: "13.2.2"

Pg. B-9 (Volumes 1 and 2) - Under Wander azimuth coordinates (defined), change "2.2" to "4.5"

Page 15-37 (Part 2):

The first IF statement under the DO loop should be changed to:

IF j is less than or equal to the specified $\Phi_{\lambda y}$ expansion order, THEN:

The indentation for the last two lines in the DO loop should be the same as for the preceding line:

IF j is equal to the specified $\Phi_{\lambda y}$ expansion order: $B = \Phi_{\lambda y} \Phi_{\lambda\lambda}^{T}$

$$M_{\lambda\lambda}^{j} = M_{\lambda\lambda}^{j-1} \Delta\Phi_{\lambda\lambda_m}$$
$$\Phi_{\lambda\lambda}^{j} = \Phi_{\lambda\lambda}^{j-1} + \frac{1}{j\,!} M_{\lambda\lambda}^{j}$$