SCHULER OSCILLATIONS

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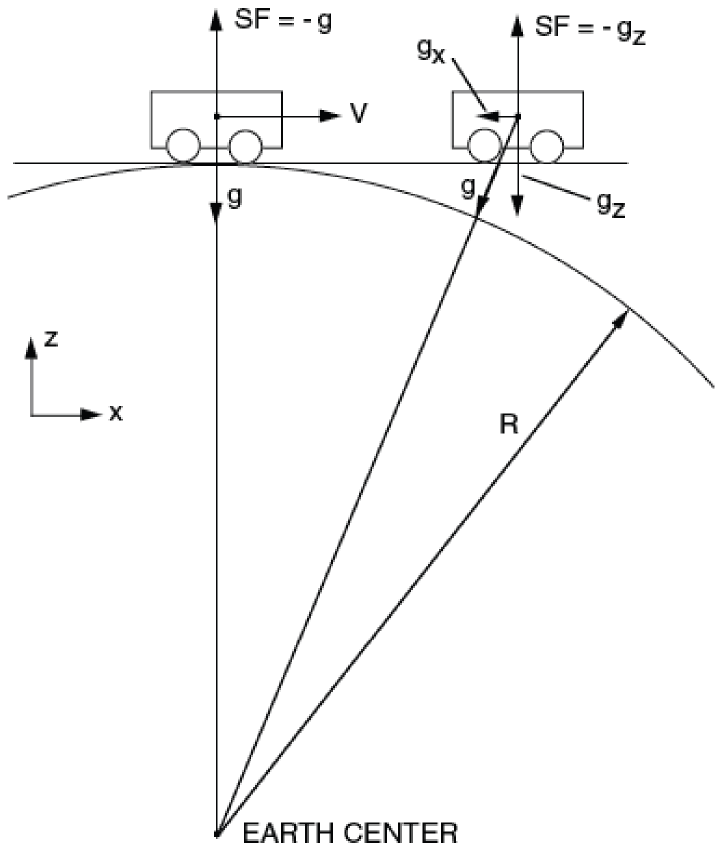
Schuler oscillations are characteristic 84 minute cyclic responses of the correct navigation calculations in an INS (inertial navigation system) to incorrect INS initialization and inertial sensor errors. The designation "Schuler Oscillations" for the 84 minute period error characteristics, is based on analogy to the behavior of an undamped pendulum having an 84 minute oscillation response to disturbances. Dr. Maximilian Schuler (a German scientist active during the early 1900's time period) reasoned that such a pendulum would remain vertical under horizontal acceleration of the suspension point. Thus, with such a pendulum, an accurate vertical reference could be established in a dynamic environment such as on a moving ship. His rationale for selecting the pendulum period was based on setting the pendulocity of the pendulum so that horizontal acceleration of the pivot over the earth surface produced angular acceleration that matched the equivalent angular acceleration of the pivot over the earth’s curved surface. Dr. Schuler demonstrated analytically that the pendulum period to make this match would be 84 minutes, i.e., the same oscillation frequency as for INS horizontal errors. Without disturbances, a Schuler pendulum would remain vertical under horizontal pivot acceleration. If disturbed (e.g., by physically touching the pendulum), a Schuler pendulum would exhibit 84 minute oscillations (the reciprocal now known as the “Schuler frequency”). Similarly, for an INS without error, a gyro derived vertical reference frame within the INS would maintain a correct vertical orientation under system acceleration. However, if the INS contained errors, the gyro derived vertical reference would then exhibit 84 minute error oscillations from the vertical (as would the accompanying INS computed horizontal velocity and position outputs). Thus, in effect, implementation of an INS creates an artificial Schuler pendulum within the INS that exhibits the same error response behavior as a “real” Schuler pendulum.

Schuler oscillations in an INS are caused by the effect of gravitational components (correctly modeled in the INS computer) that are perpendicular to the direction of motion. For example, consider the following figure depicting a vehicle rolling without friction on a horizontal plane tangent to the earth. If the vehicle contains an INS having an erroneous initial horizontal velocity V at the point of tangency, the inertial navigation computation will integrate V into a horizontal position change X which will then generate a computed gravitational component g_X opposite to the direction of motion proportional to X divided by earth's radius. As the motion continues, the horizontal gravitational acceleration will eventually stop and reverse the motion, generating an undamped



oscillation with frequency equal to the square root of earth's gravity magnitude divided by earth's radius ($\sqrt{g / R}$). This is the so-called Schuler frequency having a period of approximately 84 minutes. The motion would be undetected by the INS force-acceleration measuring instruments (i.e., accelerometers), because gravity is a natural property of space, not a force (Reference 1). It is important to recognize however, that the identical solution would be generated if the true initial velocity was V and the INS was correctly initialized at V. Thus, 84 minute type oscillations can also be generated naturally. Schuler oscillations have been classified as only those 84 minute oscillations that are generated by INS errors.

A similar analysis would show that 84 minute oscillations can be generated by true X axis acceleration (measurable by an INS X axis accelerometer), or by X axis accelerometer errors leading to Schulering. INS gyros are used to measure true rotation of the accelerometers so that measured acceleration outputs are properly angularly interpreted in INS navigation coordinates (the coordinate frame used to compute velocity and position by acceleration integration). However, gyros can also generate Schuler oscillations in an INS because when in error, they erroneously misinterpret the orientation of the accelerometers, leading to acceleration component errors.

Any analytical method for suppressing Schuler oscillations within an INS (e.g., damping) must include a method of separating the erroneous (Schuler) from the true 84 minute oscillation components. This is why other non-INS measurements of motion are traditionally included in INS error correction operations (e.g., Kalman filtering with GPS position inputs) so that only the erroneous oscillation generating INS components are isolated for suppression. It follows then that any INS computational scheme designed to suppress Schuler oscillations with no other input than the INS inertial sensors, must also incorrectly suppress true 84 minute oscillations. The result would inevitably be an INS that generates erroneous outputs under normal motion, even when containing error free inertial sensors. An example would be an aircraft in a circular holding pattern of 84 minutes per cycle which would generate 84 minute velocity and position oscillations along earth fixed horizontal axes. Other true inputs with near 84 minute frequency components would also be attenuated, the degree depending on the bandwidth of the Schuler suppression methodology used.

The fundamental problem with adding a damping feature into the basic INS inertial navigation computations is that it is difficult to hypothesize the types of true natural motion encountered that could adversely impact performance and the associated degree thereof. Thus, there would always be a degree of uncertainty in the validity of INS outputs being a dependable measure of actual motion. In contrast, for a typical unaided INS, there will always be Schuler oscillations, but their magnitude will always be limited by the accuracy of the inertial sensors being used (known specification controlled values). Thus a user can be assured that the magnitude of error will not exceed known limits regardless of the particular trajectory being followed.

It is to be noted that for inertial sensors having limited accuracy (so that they are unsuitable for inertial navigation application - e.g., 1 deg/hr accuracy gyros compared with 0.01 deg/hr INS quality requirements), damping with accelerometer deduced verticality control has been applied successfully to build attitude/heading reference systems (AHRS). Because of the large angular error rate of AHRS gyros, the angular output measurement from the gyros would build unacceptably with time. To compensate for the growing attitude error, an AHRS measures and controls the error from verticality using a low gain feedback from accelerometers as the measurement device. (Based on the approximation that the dominant output from the accelerometers will be one g force acceleration upward to balance the one g downward acceleration of gravity.) As a result, vertical reference angle outputs from an AHRS (pitch and roll) are well known to be adversely affected by continuous turning maneuvers that force the gyro derived vertical reference (with accelerometer low-gain verticality correction control) to lean into the accelerometer derived vertical (i.e., toward the net force balancing gravity and creating the turn as sensed by the accelerometers). AHRS accelerometer control loop time constants are typically set for one to two minutes response time (i.e., not 84 minutes) to still provide an accurate gyro responsive angular reference through short-term maneuvers. Users are aware of AHRS limitations and allow for verticality error during extended steady maneuvers.

In addition to velocity/position outputs, an INS also provides attitude outputs similar to an AHRS. Unlike an AHRS, however, INS users rely on outputs being trajectory independent. This was one of the many reasons that Boeing converted their commercial aircraft configurations (since introduction of the 757 in 1980) to use INS outputs for attitude control loop referencing (e.g., automatic landing); to eliminate previously used AHRS vertical references from introducing attitude errors (e.g., during final approach) (Reference 2). Most commercial aircraft manufacturers (e.g., Airbus) have since followed the same path. Unlike an AHRS, INS navigation software alteration for Schuler damping is not a practical alternative because a pilot (or autopilot) would not be capable of deducing when INS outputs might be in error from maneuver induced distortion, i.e., not recognizing when a maneuver might contain long term 84 minute cyclic components that create error through the artificial Schuler loop damping mechanism.

<u>References</u>

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[2] Savage, P.G., "Blazing Gyros: The Evolution of Strapdown Inertial Navigation Technology for Aircraft", AIAA Journal of Guidance, Control and Dynamics, Vol. 36, No. 3, May - June, 2013, pp. 637-655.