# ANALYTICALLY DERIVING HOW RING LASER AND FIBER OPTIC GYROS MEASURE ANGULAR ROTATION

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## ABSTRACT

This article provides a rigorous derivation of the analytics governing the operation of optical angular rotation sensors, ring laser gyros (RLGs) and fiber optic gyros (FOGs). Analytical predictions match known actual gyro test results, showing how angular rotation induces phase shift in oppositely directed monochromatic light waves traversing a closed waveguide, then measured by photo detectors for output. The waveguide is formed by reflecting mirrors in an RLG and by a fiber optic coil in a FOG. Classical Galilean/Newtonian kinematics explains how rotation affects each oppositely directed wave. When rotation induced tangential velocity of the waveguide is small compared to the speed of light, it is shown that Relativity theory predicts the same result. To analytically derive how the waves intersect in space-time when merged at the gyro readout, Relativity theory alone is required for the solution. The result is a single equation describing the net phase shift induced by rotation in the merged waves when illuminating the gyro readout photo detector. Applying the equation to an RLG shows how cyclic outputs are generated from the photo detector, each representing a known increment of angular rotation. Application to a FOG shows how successive angular rotation increments are measured, each over the time for a wave to traverse the fiber coil. Analysis of "closed-loop" FOG operations derives how to operate L/N (lithium–niobate) crystals inserted in the fiber coil to balance rotation induced phase shift and generate angular rotation outputs. Two closed-loop FOG configurations are analytically described, one using symmetrical dual L/N crystal inserts, the other using a single L/N crystal to reduce manufacturing cost. Included are methods for eliminating round-off error in FOG digital conversion operations.

## DISPOSITION

This article has been revised from its original version and republished as

Savage, P. G., "Analytical Description of Optical Gyros", SAI WBN-14024, Apr 3, 2019, free access available at www.strapdownassociates.com.

The new WBN-14024 version has been modified from the original for enhanced clarity and simplicity. Results and conclusions are unchanged from the original WBN-14023 version. Major revisions are in the optical gyro generalization of analytical material originally derived individually for RLGs and FOGs, comprehensively deriving the analytical relationship between

photons and light waves traversing the optical gyro waveguide under rotation, and eliminating several approximations used in the original derivation process. The original WBN-14023 version can still be accessed on line at strapdownassociates.com/How Optical Gyros Operate.pdf.