APPENDICES F, G, AND H

TO

GENERATING STRAPDOWN SPECIFIC-FORCE/ANGULAR-RATE FOR SPECIFIED ATTITUDE/ POSITION VARIATION FROM A REFERENCE TRAJECTORY

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INTRODUCTION

This document is an addendum to [1], providing derivations in Appendices F, G, and H herein deemed too detailed for inclusion in [1]. Appendix F provides useful formulas applied in Appendices G and H, Appendix G provides derivations of specific-force/velocity/position solutions generated using the Method 1approach under [1] defined test example conditions, and Appendix H provides derivations of specific-force/velocity/position solutions generated using Method 2 under the same defined test example conditions. Definitions for analytical parameters appearing in these appendices are provided in [1] and repeated where used in this addendum.

NOTATION

General Notation

$\underline{V}$ = Arbitrary vector without specific coordinate frame component definition.

$\underline{V}^A$ = Column matrix with elements equal to general vector $\underline{V}$ projections on general coordinate frame A axes.

$\left(\underline{V}^A \times\right)$ = Cross-product (or skew symmetric) form of $\underline{V}^A$ defined such that for the cross-product of $\underline{V}$ with another arbitrary vector $\underline{W}$ in the general A frame: $\underline{V}^A \times \underline{W}^A = \left(\underline{V}^A \times\right) \underline{W}^A$.

C_A^D = Generalized direction cosine matrix that transforms vectors from general coordinate frame A to general coordinate frame D (i.e., $\underline{V}^D = C_A^D \underline{V}^A$).

Coordinate Frames

B = "Body" coordinate frame aligned with orthogonal strapdown inertial sensor axes fixed in the rotating body.

E = Earth frame fixed to the rotating earth.

E_0 = Inertial non-rotating inertial frame aligned with E at trajectory start time t = 0.

Trajectory Generator Update Cycle Indices

m = Trajectory generator update cycle index ($m = 0$ at trajectory start time t = 0).
n = Trajectory generator even (or alternate) update cycle index (i.e., $m = 2n$).
Important Note: Each cycle index **subscript** identifies the ***m* cycle** time instant value for that parameter (e.g., subscript 2n indicates a parameter value as cycle $m = 2n$, and 2n-1 indicates a parameter value at cycle $m = 2n - 1$.

Trajectory Type Subscripts

ref = Parameter or coordinate frame identifier for the variation trajectory.
var = Parameter or coordinate frame identifier for the variation trajectory.

Parameter Definitions

Parameters are listed next in alphabetical order with Greek letters ordered using the English translation (i.e., Delta Δ under D, mu μ under m, omega ω under o, phi ϕ under p, upsilon υ under u). Parameters used exclusively in the appendices are defined separately in the appendices where they appear.

$\underline{a}_{SF}^{B}$ = Specific force acceleration vector of the rotating body (that would be measured by strapdown accelerometers attached to the rotating body and aligned with body axes).
$C_{A_m}^{D}$ = Direction cosine matrix C_A^D at the end of trajectory update cycle m.
$\Delta\underline{\alpha}_m^B$ = Integral over an m cycle of B frame measured inertial angular rate $\underline{\omega}^B$ (that would be measured by strapdown gyros attached to the rotating body and aligned with body axes, i.e., $\Delta\underline{\alpha}_m^B = \int_{t_{m-1}}^{t_m} \underline{\omega}^B dt$.
$\Delta\underline{\alpha}_{var}^{Bvar}$ = Particular value of $\Delta\underline{\alpha}_m^B$ defined for the sample to be <u>constant</u> for $0 \geq m > -9$.
$\Delta\underline{\alpha}'^{Bvar}_{var}$ = Particular value of $\Delta\underline{\alpha}_m^B$ defined for the sample to be <u>constant</u> for $m > 0$.
$\Delta\underline{\upsilon}_m^B$ = Integral over an m cycle of B frame measured specific-force (acceleration) $\underline{a}_{SF}^B$ (i.e., $\Delta\underline{\upsilon}_m^B = \int_{t_{m-1}}^{t_m} \underline{a}_{SF}^B \, dt$).
$\underline{g}$ = Earth's mass attraction gravity vector (relative to earth's center) at trajectory position location $\underline{R}$.
$\underline{g}_{avg}$ = Constant average approximation of $\underline{g}$ to simplify the test example model.
I = Identity matrix.
$\underline{l}^{Bref}$ = Specified position displacement vector of $\underline{R}_{var}$ relative to $\underline{R}_{ref}$, a constant in the $Bref$ frame for the test example.

$\underline{\omega}^B$ = Rotating body angular rate vector relative to non-rotating inertial space that would be measured by strapdown gyros attached to the body and aligned with rotating body axes.
$\underline{R}$ = Position vector from earth's center to the trajectory designated position location ("navigation center").
$\underline{R}_m$ = Position vector $\underline{R}$ at the end of trajectory update cycle *m*.
t = Elapsed time from the start of a trajectory.
t_m = Time *t* at the end of trajectory update cycle *m*.
T_m = Time interval from t_{m-1} to t_m (assumed constant for this article).
$\underline{V}$ = Velocity of trajectory position relative to non-rotating inertial space defined as the time rate of change of position evaluated in inertially non-rotating E_0 coordinates: $\underline{V}^{E0} \equiv \frac{d\,\underline{R}^{E0}}{dt}$.
$\underline{V}_m$ = Velocity vector $\underline{V}$ at the end of trajectory update cycle *m*.

APPENDIX F

USEFUL FORMULAS

First order expansion approximations are employed in Appendix G and H for matrices of the form $M = \left[I + a\; E_a \right]$ where *I* is the identity matrix *a* is an arbitrary scalar and matrix E_a is much smaller than *I*. Additionally, note that $\left[I + a\; E_a \right]^{-1} \left[I + a\; E_a \right] = I$ and that $\left[I - a\; E_a \right] \left[I + a\; E_a \right] + a^2\, E_a\, E_a = I$. Equating the previous two expressions and multiplying on the right by $\left[I + a\; E_a \right]^{-1}$ yields the useful formula:

$$\left[I + a\; E_a \right]^{-1} = \left[I - a\; E_a \right] + a^2\, E_a\, E_a \left[I + a\; E_a \right]^{-1} \approx \left[I - a\; E_a \right] \qquad \text{(F-1)}$$

Another useful identity derived in [?, Sect. 3.1.1] is [?, Eq. (3.1.1-38)] is:

$$C \left(\underline{V} \times \right) C^{-1} = \left[\left(C\, \underline{V} \right) \times \right] \qquad \text{(F-2)}$$

where *C* is an arbitrary direction cosine matrix, $\underline{V}$ is an arbitrary vector, and $\left(\underline{V} \times \right)$ is the cross-product skew-symmetric form of $\underline{V}$ as defined in the Notation section of this article.

APPLICABLE EQUATIONS FROM THE MAIN ARTICLE

The following equations taken from the main article are referenced in Appendices G and H to follow, and repeated next for convenient referencing.

$$G_{Vvar_m} \equiv I + \frac{1-\cos\Delta\alpha_{var_m}^{Bvar}}{\left(\Delta\alpha_{var_m}^{Bvar}\right)^2}\left(\Delta\underline{\alpha}_{var_m}^{Bvar}\times\right) + \frac{1}{\Delta\alpha_{var_m}^2}\left(1-\frac{\sin\Delta\alpha_{var_m}^{Bvar}}{\Delta\alpha_{var_m}^{Bvar}}\right)\left(\Delta\underline{\alpha}_{var_m}^{Bvar}\times\right)^2$$

$$\Delta\underline{\upsilon}_{var_m}^{Bvar} \equiv \int_{t_{m-1}}^{t_m} \underline{a}_{SF_{var}}^{Bvar}\, dt \qquad (3)$$

$$\underline{V}_{var_m}^{E0} = \underline{V}_{var_{m-1}}^{E0} + C_{Bvar_{m-1}}^{E0}\, G_{Vvar_m}\, \Delta\underline{\upsilon}_{var_m}^{Bvar} + \frac{1}{2}\left(\underline{g}_{var_m}^{E0} + \underline{g}_{var_{m-1}}^{E0}\right) T_m$$

$$G_{Rvar_m} \equiv \frac{1}{2} I + \frac{1}{\left(\Delta\alpha_{var_m}^{Bvar}\right)^2}\left(1-\frac{\sin\Delta\alpha_{var_m}^{Bvar}}{\Delta\alpha_{var_m}^{Bvar}}\right)\left(\Delta\underline{\alpha}_{var_m}^{Bvar}\times\right)$$

$$+\frac{1}{\left(\Delta\alpha_{var_m}^{Bvar}\right)^2}\left[\frac{1}{2}-\left(\frac{1-\cos\Delta\alpha_{var_m}^{Bvar}}{\left(\Delta\alpha_{var_m}^{Bvar}\right)^2}\right)\right]\left(\Delta\underline{\alpha}_{var_m}^{Bvar}\times\right)^2 \qquad (4)$$

$$\underline{R}_{var_m}^{E0} = \underline{R}_{var_{m-1}}^{E0} + \underline{V}_{var_{m-1}}^{E0}\, T_m + C_{Bvar_{m-1}}^{E0}\, G_{Rvar_m}\, \Delta\underline{\upsilon}_{var_m}^{Bvar}\, T_m + \frac{1}{6}\left(\underline{g}_{var_m}^{E0} + 2\,\underline{g}_{var_{m-1}}^{E0}\right) T_m^2$$

$$\Delta\underline{\upsilon}_{var_m}^{Bvar} = \left(C_{Bvar_{m-1}}^{E0}\, G_{Rvar_m}\right)^{-1}\left[\begin{array}{c}\underline{R}_{var_m}^{E0} - \underline{R}_{var_{m-1}}^{E0} - \underline{V}_{var_{m-1}}^{E0}\, T_m \\ -\frac{1}{6}\left(\underline{g}_{var_m}^{E0} + 2\,\underline{g}_{var_{m-1}}^{E0}\right) T_m^2\end{array}\right] / T_m \qquad (9)$$

$$A_{2n-1} \equiv C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} \left(C^{E0}_{Bvar_{2n-1}} G_{Vvar_{2n}} \right)^{-1}$$

$$B_{den_{2n-1}} \equiv \left[C^{E0}_{Bvar_{2n-2}} \left(G_{Rvar_{2n-1}} + G_{Vvar_{2n-1}} \right) - A_{2n-1} C^{E0}_{Bvar_{2n-2}} G_{Vvar_{2n-1}} \right] T_m$$

$$B_{num_{2n-1}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\, \underline{V}^{E0}_{var_{2n-2}} T_m - A_{2n-1} \left(\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}} \right) T_m$$

$$+ \left[\left(\frac{A_{2n-1}}{2} - \frac{1}{6} I \right) \underline{g}^{E0}_{var_{2n}} + \left(A_{2n-1} - I \right) \underline{g}^{E0}_{var_{2n-1}} + \left(\frac{A_{2n-1}}{2} - \frac{5}{6} I \right) \underline{g}^{E0}_{var_{2n-2}} \right] T_m^2$$

$$\Delta \underline{\upsilon}^{Bvar}_{var_{2n-1}} = B^{-1}_{den_{2n-1}} B_{num_{2n-1}}$$

(31)

$$A_{2n} \equiv C^{E0}_{Bvar_{2n-2}} \left(G_{Rvar_{2n-1}} + G_{Vvar_{2n-1}} \right) \left(C^{E0}_{Bvar_{2n-2}} G_{Vvar_{2n-1}} \right)^{-1}$$

$$B_{den_{2n}} \equiv \left(C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} - A_{2n} C^{E0}_{Bvar_{2n-1}} G_{Vvar_{2n}} \right) T_m$$

$$B_{num_{2n}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\, \underline{V}^{E0}_{var_{2n-2}} T_m - A_{2n} \left(\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}} \right) T_m$$

$$+ \left[\left(\frac{A_{2n}}{2} - \frac{1}{6} I \right) \underline{g}^{E0}_{var_{2n}} + \left(A_{2n} - I \right) \underline{g}^{E0}_{var_{2n-1}} + \left(\frac{A_{2n}}{2} - \frac{5}{6} I \right) \underline{g}^{E0}_{var_{2n-2}} \right] T_m^2$$

$$\Delta \underline{\upsilon}^{Bvar}_{var_{2n}} = B^{-1}_{den_{2n}} B_{num_{2n}}$$

For $m \le -9$: $\quad C^{E0}_{Bvar_m} = C^{E0}_{Bvar_{-9}}$

For $0 \ge m > -9$:

$$C^{E0}_{Bvar_m} \approx C^{E0}_{Bvar_{m-1}} \left[I + \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) + \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right] \approx C^{E0}_{Bvar_{m-1}} \left[I + \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] \qquad (32)$$

For $m > 0$:

$$C^{E0}_{Bvar_m} \approx C^{E0}_{Bvar_{m-1}} \left[I + \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) + \frac{1}{2} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^2 \right] \approx C^{E0}_{Bvar_{m-1}} \left[I + \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right]$$

For $m \le -9$: $\quad G_{Vvar_m} = I \qquad G_{Rvar_m} = \frac{1}{2} I$

For $0 \ge m > -9$: $\quad G_{Vvar_m} \approx I + \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \qquad G_{Rvar_m} \approx \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right]$ (33)

For $m > 0$: $\quad G_{Vvar_m} \approx I + \frac{1}{2} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \qquad G_{Rvar_m} \approx \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right]$

$$\underline{R}^{E0}_{var_m} = m\, \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_m} \underline{l}^{Bref} \qquad (37)$$

$$\underline{g}^{E0}_{var_m} \approx \text{Constant} \equiv \underline{g}^{E0}_{avg} \tag{38}$$

APPENDIX G

ANALYTICAL DETAIL FOR METHOD 1 TEST EXAMPLE SOLUTION

This appendix provides analytical detail leading to the Method 1 test example solutions of (9) for specific force and resulting velocity, position thereof. First order expansion approximations are employed in the development, facilitated by approximation methods derived in the previous Appendix F.

Under the (37) and (38) example conditions, Method 1 specific force in (9) becomes

$$\Delta\underline{\upsilon}^{Bvar}_{var_m} = \left(C^{E0}_{Bvar_{m-1}} G_{Rvar_m}\right)^{-1}\left(\underline{R}^{E0}_{var_m} - \underline{R}^{E0}_{var_{m-1}} - \underline{V}^{E0}_{var_{m-1}} T_m - \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2\right) / T_m$$

$$= \left(C^{E0}_{Bvar_{m-1}} G_{Rvar_m}\right)^{-1}\begin{pmatrix} m\,\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar_m}\,\underline{l}^{Bref} - (m-1)\,\underline{V}^{E0}_{ref_0}\, T_m \\ -\,C^{E0}_{Bvar_{m-1}}\,\underline{l}^{Bref} - \underline{V}^{E0}_{var_{m-1}} T_m - \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2 \end{pmatrix} / T_m \tag{G-1}$$

$$= \left(C^{E0}_{Bvar_{m-1}} G_{Rvar_m}\right)^{-1}\left[\left(C^{E0}_{Bvar_m} - C^{E0}_{Bvar_{m-1}}\right)\underline{l}^{Bref} - \left(\underline{V}^{E0}_{var_{m-1}} - \underline{V}^{E0}_{ref_0}\right) T_m - \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2\right] / T_m$$

Substituting (G-1) in (3) with (38) for gravity then obtains for velocity $\underline{V}^{E0}_{var_m}$:

$$\underline{V}^{E0}_{var_m} = \underline{V}^{E0}_{var_{m-1}} + C^{E0}_{Bvar_{m-1}} G_{Vvar_m}\,\Delta\underline{\upsilon}^{Bvar}_{var_m} + \underline{g}^{E0}_{avg} T_m$$

$$= \underline{V}^{E0}_{var_{m-1}} + C^{E0}_{Bvar_{m-1}} G_{Vvar_m}\left(C^{E0}_{Bvar_{m-1}} G_{Rvar_m}\right)^{-1}\begin{bmatrix} \left(C^{E0}_{Bvar_m} - C^{E0}_{Bvar_{m-1}}\right)\dfrac{\underline{l}^{Bref}}{T_m} \\ -\left(\underline{V}^{E0}_{var_{m-1}} - \underline{V}^{E0}_{ref_0}\right) - \frac{1}{2}\underline{g}^{E0} T_m \end{bmatrix} + \underline{g}^{E0}_{avg} T_m$$

$$= \left[I - C^{E0}_{Bvar_{m-1}} G_{Vvar_m}\,\left(G_{Rvar_m}\right)^{-1}\left(C^{E0}_{Bvar_{m-1}}\right)^{-1}\right]\underline{V}^{E0}_{var_{m-1}} \tag{G-2}$$

$$+\,C^{E0}_{Bvar_{m-1}} G_{Vvar_m}\,\left(G_{Rvar_m}\right)^{-1}\left(C^{E0}_{Bvar_{m-1}}\right)^{-1}\begin{bmatrix} \left(C^{E0}_{Bvar_m} - C^{E0}_{Bvar_{m-1}}\right)\dfrac{\underline{l}^{Bref}}{T_m} \\ +\,\underline{V}^{E0}_{ref_0} - \frac{1}{2}\underline{g}^{E0} T_m \end{bmatrix} + \underline{g}^{E0}_{avg} T_m$$

With (32), the $\left(C^{E0}_{Bvar_m} - C^{E0}_{Bvar_{m-1}}\right)$ term in (G-1) and (G-2) is to first order:

$$\text{For } m \le -9:$$
$$C^{E0}_{Bvar_m} - C^{E0}_{Bvar_{m-1}} = 0$$
$$\text{For } 0 \ge m > -9:$$
$$C^{E0}_{Bvar_m} - C^{E0}_{Bvar_{m-1}} \approx C^{E0}_{Bvar_{m-1}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right] - C^{E0}_{Bvar_{m-1}} = C^{E0}_{Bvar_{m-1}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) \tag{G-3}$$
$$\text{For } m > 0:$$
$$C^{E0}_{Bvar_m} - C^{E0}_{Bvar_{m-1}} \approx C^{E0}_{Bvar_{m-1}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)$$

Applying (33) and (F-1) of Appendix F, the $G_{Vvar_m}\left(G_{Rvar_m}\right)^{-1}$ term in (G-2) is to first order accuracy:

$$\text{For } m \le -9:$$
$$G_{Vvar_m}\left(G_{Rvar_m}\right)^{-1} = I\left(\frac{1}{2}I\right)^{-1} = 2I$$
$$\text{For } 0 \ge m > -9:$$
$$G_{Vvar_m}\left(G_{Rvar_m}\right)^{-1} \approx \left[I + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left\{\frac{1}{2}\left[I + \frac{1}{3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\right\}^{-1} \tag{G-4}$$
$$\approx 2\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left[I - \frac{1}{3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right] \approx 2\left[I + \left(\frac{1}{2} - \frac{1}{3}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right] = 2\left[I + \frac{1}{6}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]$$
$$\text{For } m > 0:$$
$$G_{Vvar_m}\left(G_{Rvar_m}\right)^{-1} \approx 2\left[I + \frac{1}{6}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]$$

With (G-4) and (F-2) of Appendix F, the $C^{E0}_{Bvar_{m-1}} G_{Vvar_m}\left(G_{Rvar_m}\right)^{-1}\left(C^{E0}_{Bvar_{m-1}}\right)^{-1}$ term is given by

$$\text{For } m \le -9:$$

$$C_{Bvar_{m-1}}^{E0}\, G_{Vvar_m} \left(G_{Rvar_m}\right)^{-1} \left(C_{Bvar_{m-1}}^{E0}\right)^{-1} = 2I$$

$$\text{For } 0 \ge m > -9:$$

$$C_{Bvar_{m-1}}^{E0}\, G_{Vvar_m} \left(G_{Rvar_m}\right)^{-1} \left(C_{Bvar_{m-1}}^{E0}\right)^{-1} = 2\, C_{Bvar_{m-1}}^{E0} \left[I + \frac{1}{6} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times\right)\right] \left(C_{Bvar_{m-1}}^{E0}\right)^{-1} \quad \text{(G-5)}$$

$$= 2I + \frac{1}{3}\, C_{Bvar_{m-1}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times\right) \left(C_{Bvar_{m-1}}^{E0}\right)^{-1} = 2I + \frac{1}{3}\left[\left(C_{Bvar_{m-1}}^{E0}\, \Delta \underline{\alpha}_{var}^{Bvar}\right) \times\right]$$

$$\text{For } m > 0:$$

$$C_{Bvar_{m-1}}^{E0}\, G_{Vvar_m} \left(G_{Rvar_m}\right)^{-1} \left(C_{Bvar_{m-1}}^{E0}\right)^{-1} = 2I + \frac{1}{3}\left[\left(C_{Bvar_{m-1}}^{E0}\, \Delta \underline{\alpha}'^{Bvar}_{var}\right) \times\right]$$

Substituting (G-3) – (G-5) in (G-2) and retaining only first order terms then obtains velocity $\underline{V}_{var_m}^{E0}$ generated by Method 1 under the test example conditions:

$$\text{For } m \le -9: \quad \underline{V}_{var_m}^{E0} = \underline{V}_{ref_0}^{E0}$$

$$\text{For } m = -8, -6, -4, -2, 0:$$

$$\underline{V}_{var_m}^{E0} \approx \underline{V}_{ref_0}^{E0} + 2\, C_{Bvar_{m-1}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m}\right) - \frac{1}{6}\left(C_{Bvar_{m-1}}^{E0}\, \Delta \underline{\alpha}_{var}^{Bvar}\right) \times \underline{g}_{avg}^{E0}\, T_m$$

$$\text{For } m = -7, -5, -3, -1: \quad \underline{V}_{var_m}^{E0} = \underline{V}_{ref_0}^{E0}$$

$$\text{For } m = 1, 3, 5, \cdots: \qquad \text{(G-6)}$$

$$\underline{V}_{var_m}^{E0} = \underline{V}_{ref_0}^{E0} + 2\, C_{Bvar_{m-1}}^{E0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar}\right) \times \frac{\underline{l}^{Bref}}{T_m}\right]$$

$$- \frac{1}{6}\left[C_{Bvar_{m-1}}^{E0} \left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar}\right)\right] \times \underline{g}_{avg}^{E0}\, T_m$$

$$\text{For } m = 2, 4, 6, \cdots:$$

$$\underline{V}_{var_m}^{E0} = \underline{V}_{ref_0}^{E0} + 2\, C_{Bvar_{m-1}}^{E0} \left[\Delta \underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m}\right] - \frac{1}{6}\left(C_{Bvar_{m-1}}^{E0}\, \Delta \underline{\alpha}_{var}^{Bvar}\right) \times \underline{g}_{avg}^{E0}\, T_m$$

Derivation steps that led to (G-6) are discussed at the end of this appendix.

Substituting (G-6) into (G-1) with (33) for G_{Rvar_m} yields, to first order accuracy, the corresponding Method 1specific force profile that generated the (G-6) velocity response:

$$\text{For } m \le -9: \quad \Delta\underline{\upsilon}_{var_m}^{Bvar} = -\left(C_{Bvar_{-9}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0}\, T_m$$

$$\text{For } m = -8, -6, -4, -2, 0:$$

$$\Delta\underline{\upsilon}_{var_m}^{Bvar} \approx 2\,\Delta\underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} - \left[I - \frac{1}{3}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times\right)\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0}\, T_m$$

$$\text{For } m = -7, -5, -3, -1:$$

$$\Delta\underline{\upsilon}_{var_m}^{Bvar} = -2\,\Delta\underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} - \left[I - \frac{2}{3}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times\right)\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0}\, T_m$$

$$\text{For } m = 1, 3, 5, \cdots: \qquad \text{(G-7)}$$

$$\Delta\underline{\upsilon}_{var_m}^{Bvar} = 2\left(\Delta\underline{\alpha}'^{Bvar}_{var} - 2\,\Delta\underline{\alpha}_{var}^{Bvar}\right) \times \frac{\underline{l}^{Bref}}{T_m}$$

$$- \left\{ I - \frac{1}{3}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\right\}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0}\, T_m$$

$$\text{For } m = 2, 4, 6, \cdots:$$

$$\Delta\underline{\upsilon}_{var_m}^{Bvar} = -2\left(\Delta\underline{\alpha}'^{Bvar}_{var} - 2\,\Delta\underline{\alpha}_{var}^{Bvar}\right) \times \frac{\underline{l}^{Bref}}{T_m}$$

$$- \left\{ I - \frac{1}{3}\left[\left(2\,\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\right\}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0}\, T_m$$

Derivation steps that led to (G-7) are discussed at the end of this appendix.

<u>Velocity Derivation Steps Leading To (G-6):</u>

Applying (G-3) – (G-5) in (G-2) and retaining only first order terms obtains velocity $\underline{V}_{var_m}^{E0}$ generated by Method 1 under test example conditions for *m* cycles -8 and lower:

$$\text{For } m < -8: \quad \underline{V}^{E0}_{var_{-9}} = \underline{V}^{E0}_{ref_0}$$

$$\text{For } m = -8:$$

$$\underline{V}^{E0}_{var_{-8}} = \left[I - C^{E0}_{Bvar_{-9}} G_{Vvar_{-8}} \left(G_{Rvar_{-8}}\right)^{-1} \left(C^{E0}_{Bvar_{-9}}\right)^{-1} \right] \underline{V}^{E0}_{var_{-9}}$$

$$+ C^{E0}_{Bvar_{-9}} G_{Vvar_{-8}} \left(G_{Rvar_{-8}}\right)^{-1} \left(C^{E0}_{Bvar_{-9}}\right)^{-1} \left[\begin{array}{c} \left(C^{E0}_{Bvar_{-8}} - C^{E0}_{Bvar_{-9}}\right) \dfrac{\underline{l}^{Bref}}{T_m} \\ + \underline{V}^{E0}_{ref_0} - \dfrac{1}{2} \underline{g}^{E0} T_m \end{array} \right] + \underline{g}^{E0}_{avg} T_m$$

$$= -\left[I + \frac{1}{3} \left[\left(C^{E0}_{Bvar_{-9}} \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \right] \underline{V}^{E0}_{ref_0} \qquad \text{(G-8)}$$

$$+ \left\{ 2I + \frac{1}{3} \left[\left(C^{E0}_{Bvar_{-9}} \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \right\} \left[C^{E0}_{Bvar_{-9}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) + \underline{V}^{E0}_{ref_0} - \frac{1}{2} \underline{g}^{E0}_{avg} T_m \right] + \underline{g}^{E0}_{avg} T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + 2\, C^{E0}_{Bvar_{-9}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) - \frac{1}{6} \left(C^{E0}_{Bvar_{-9}} \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \underline{g}^{E0}_{avg} T_m$$

The next velocity calculations use the following first order approximation development based on $C^{E0}_{Bvar_m}$ in (32) multiplied by $\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) \right]^{-1}$ with (F-1) from Appendix F.

$$C^{E0}_{Bvar_{m-1}} = C^{E0}_{Bvar_m} \left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) \right]^{-1} \approx C^{E0}_{Bvar_m} \left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) \right]$$

$$\therefore \; C^{E0}_{Bvar_{m-1}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) = C^{E0}_{Bvar_m} \left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) \right] \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) \approx C^{E0}_{Bvar_m} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) \qquad \text{(G-9)}$$

$$\text{Similarly:} \quad C^{E0}_{Bvar_{m-1}} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \approx C^{E0}_{Bvar_m} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right)$$

Eq. (G-9) is used frequently in this appendix for Method 1 velocity derivations and in Appendix H for the Method 2 solution under test example conditions.

Applying (G-8) - (G-9) and (G-3) – (G-5) in (G-2) finds for $m = -7$:

For $m = -7$:

$$\underline{V}^{E0}_{var_{-7}} = \left[I - C^{E0}_{Bvar_{-8}} G_{Vvar_{-7}} \left(G_{Rvar_{-7}}\right)^{-1} \left(C^{E0}_{Bvar_{-8}}\right)^{-1} \right] \underline{V}^{E0}_{var_{-8}}$$

$$+ C^{E0}_{Bvar_{-8}} G_{Vvar_{-7}} \left(G_{Rvar_{-7}}\right)^{-1} \left(C^{E0}_{Bvar_{-8}}\right)^{-1} \left[\begin{array}{c} \left(C^{E0}_{Bvar_{-7}} - C^{E0}_{Bvar_{-8}}\right) \dfrac{\underline{l}^{Bref}}{T_m} \\ + \underline{V}^{E0}_{ref_0} - \dfrac{1}{2} \underline{g}^{E0}_{avg} T_m \end{array} \right] + \underline{g}^{E0}_{avg} T_m$$

$$= -\left[I + \frac{1}{3}\left[\left(C^{E0}_{Bvar_{-8}} \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right] \left[\begin{array}{c} \underline{V}^{E0}_{ref_0} + 2\, C^{E0}_{Bvar_{-9}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \dfrac{\underline{l}^{Bref}}{T_m} \right) \\ -\dfrac{1}{6}\left(C^{E0}_{Bvar_{-9}} \Delta\underline{\alpha}^{Bvar}_{var}\right) \times \underline{g}^{E0}_{avg} T_m \end{array} \right] \tag{G-10}$$

$$+ \left\{ 2I + \frac{1}{3}\left[\left(C^{E0}_{Bvar_{-8}} \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \right\} \left[C^{E0}_{Bvar_{-9}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) + \underline{V}^{E0}_{ref_0} - \frac{1}{2} \underline{g}^{E0}_{avg} T_m \right] + \underline{g}^{E0}_{avg} T_m$$

$$\approx -\underline{V}^{E0}_{ref_0} - 2\, C^{E0}_{Bvar_{-8}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) + \frac{1}{6} \left(C^{E0}_{Bvar_{-8}} \Delta\underline{\alpha}^{Bvar}_{var}\right) \times \underline{g}^{E0}_{avg} T_m$$

$$-\frac{1}{3}\left(C^{E0}_{Bvar_{-8}} \Delta\underline{\alpha}^{Bvar}_{var}\right) \times \underline{V}^{E0}_{ref_0} + 2\, C^{E0}_{Bvar_{-8}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) + 2\, \underline{V}^{E0}_{ref_0}$$

$$- \underline{g}^{E0} T_m + \frac{1}{3}\left(C^{E0}_{Bvar_{-8}} \Delta\underline{\alpha}^{Bvar}_{var}\right) \times \underline{V}^{E0}_{ref_0} - \frac{1}{6}\left(C^{E0}_{Bvar_{-8}} \Delta\underline{\alpha}^{Bvar}_{var}\right) \times \underline{g}^{E0} T_m + \underline{g}^{E0}_{avg} T_m$$

$$= \underline{V}^{E0}_{ref_0}$$

Similarly, the velocity solution for $m = -6$ through $m = 0$ is as previously derived for $m = -8$ and -7 in (G-8) and (G-10).

Following the procedure leading to (G-10) then obtains for *m* cycles 1 and 2:

For $m = 1$:

$$\underline{V}^{E0}_{var_1} = \left[I - C^{E0}_{Bvar_0} \, G_{Vvar_1} \, \left(G_{Rvar_1}\right)^{-1} \left(C^{E0}_{Bvar_0}\right)^{-1} \right] \underline{V}^{E0}_{var_0}$$

$$+ C^{E0}_{Bvar_0} \, G_{Vvar_1} \, \left(G_{Rvar_1}\right)^{-1} \left(C^{E0}_{Bvar_0}\right)^{-1} \left[\begin{array}{c} \left(C^{E0}_{Bvar_1} - C^{E0}_{Bvar_0}\right) \dfrac{\underline{l}^{Bref}}{T_m} \\ + \underline{V}^{E0}_{ref_0} - \dfrac{1}{2} \underline{g}^{E0}_{avg} \, T_m \end{array} \right] + \underline{g}^{E0}_{avg} \, T_m$$

$$= - \left[I + \frac{1}{3} \left[\left(C^{E0}_{Bvar_0} \, \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \right] \right] \left[\begin{array}{c} \underline{V}^{E0}_{ref_0} + 2 \, C^{E0}_{Bvar_{-1}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \dfrac{\underline{l}^{Bref}}{T_m} \right) \\ - \dfrac{1}{6} \left(C^{E0}_{Bvar_{-1}} \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \underline{g}^{E0}_{avg} \, T_m \end{array} \right] \qquad \text{(G-11)}$$

$$+ \left\{ 2I + \frac{1}{3} \left[\left(C^{E0}_{Bvar_0} \, \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \right] \right\} \left[C^{E0}_{Bvar_0} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) + \underline{V}^{E0}_{ref_0} - \frac{1}{2} \underline{g}^{E0}_{avg} \, T_m \right] + \underline{g}^{E0}_{avg} \, T_m$$

$$\approx - \underline{V}^{E0}_{ref_0} - 2 \, C^{E0}_{Bvar_0} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) + \frac{1}{6} \left(C^{E0}_{Bvar_0} \, \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \underline{g}^{E0}_{avg} \, T_m$$

$$- \frac{1}{3} \left(C^{E0}_{Bvar_0} \, \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \underline{V}^{E0}_{ref_0} + 2 \, C^{E0}_{Bvar_0} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) + 2 \, \underline{V}^{E0}_{ref_0}$$

$$- \underline{g}^{E0} T_m + \frac{1}{3} \left(C^{E0}_{Bvar_0} \, \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \underline{V}^{E0}_{ref_0} - \frac{1}{6} \left(C^{E0}_{Bvar_0} \, \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \underline{g}^{E0} T_m + \underline{g}^{E0}_{avg} \, T_m$$

$$= \underline{V}^{E0}_{ref_0} + 2 \, C^{E0}_{Bvar_0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \frac{\underline{l}^{Bref}}{T_m} \right] - \frac{1}{6} \, C^{E0}_{Bvar_0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \underline{g}^{E0} T_m \right]$$

For $m = 2$:

$$\underline{V}_{var_2}^{E_0} = \left[I - C_{Bvar_1}^{E_0} G_{Vvar_2} \left(G_{Rvar_2} \right)^{-1} \left(C_{Bvar_1}^{E_0} \right)^{-1} \right] \underline{V}_{var_1}^{E_0}$$

$$+ C_{Bvar_1}^{E_0} G_{Vvar_2} \left(G_{Rvar_2} \right)^{-1} \left(C_{Bvar_1}^{E_0} \right)^{-1} \begin{bmatrix} \left(C_{Bvar_2}^{E_0} - C_{Bvar_1}^{E_0} \right) \dfrac{\underline{l}^{Bref}}{T_m} \\ + \underline{V}_{ref_0}^{E_0} - \dfrac{1}{2} \underline{g}_{avg}^{E_0} T_m \end{bmatrix} + \underline{g}_{avg}^{E_0} T_m$$

$$= - \left[I + \frac{1}{3} \left[\left(C_{Bvar_1}^{E_0} \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \right] \right] \begin{bmatrix} \underline{V}_{ref_0}^{E_0} + 2\, C_{Bvar_0}^{E_0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \dfrac{\underline{l}^{Bref}}{T_m} \right] \\ - \dfrac{1}{6} C_{Bvar_0}^{E_0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \underline{g}_{avg}^{E_0} T_m \right] \end{bmatrix} \qquad \text{(G-12)}$$

$$+ \left\{ 2I + \frac{1}{3} \left[\left(C_{Bvar_1}^{E_0} \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \right] \right\} \left[C_{Bvar_1}^{E_0} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) + \underline{V}_{ref_0}^{E_0} - \frac{1}{2} \underline{g}_{avg}^{E_0} T_m \right] + \underline{g}_{avg}^{E_0} T_m$$

$$\approx - \underline{V}_{ref_0}^{E_0} - 2\, C_{Bvar_1}^{E_0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \frac{\underline{l}^{Bref}}{T_m} \right] + \frac{1}{6} C_{Bvar_1}^{E_0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \underline{g}_{avg}^{E_0} T_m \right]$$

$$- \frac{1}{3} \left(C_{Bvar_1}^{E_0} \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \underline{V}_{ref_0}^{E_0} + 2\, C_{Bvar_1}^{E_0} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) + 2\, \underline{V}_{ref_0}^{E_0}$$

$$- \underline{g}_{avg}^{E_0} T_m + \frac{1}{3} \left(C_{Bvar_1}^{E_0} \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \underline{V}_{ref_0}^{E_0} - \frac{1}{6} \left(C_{Bvar_1}^{E_0} \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \underline{g}^{E_0} T_m + \underline{g}_{avg}^{E_0} T_m$$

$$= \underline{V}_{ref_0}^{E_0} + 2\, C_{Bvar_1}^{E_0} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} \right) - \frac{1}{6} C_{Bvar_1}^{E_0} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \underline{g}_{avg}^{E_0} T_m \right)$$

Similarly, the velocity solution for $m > 2$ is as previously derived for $m = 1$ and 2 in (G-11) and (G-12). The combined result for $\underline{V}_{var_m}^{E_0}$ velocity is then as shown in (G-6).

Specific Force Derivation Steps Leading To (G-7):

Included in the following development for specific force is the observation from (33) with (H-2) that to first order accuracy, $\left(G_{Rvar_m} \right)^{-1} \approx 2 \left[I - \frac{1}{3} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right]$ for $0 \geq m > -9$, and $\left(G_{Rvar_m} \right)^{-1} \approx 2 \left[I - \frac{1}{3} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right]$ for $m > 0$. Applying (G-3) – (G-6) in (G-1) and retaining only first order terms then obtains specific force $\Delta \underline{\upsilon}_{var_m}^{Bvar}$ generated by Method 1 under test example conditions for eac m cycle:

For $m < -8$:

$$\Delta \underline{\upsilon}_{var_m}^{Bvar} = -\left(C_{Bvar_{m-1}}^{E0} G_{Rvar_m} \right)^{-1} \left(\underline{V}_{var_{m-1}}^{E0} - \underline{V}_{ref_0}^{E0} + \frac{1}{2} \underline{g}_{avg}^{E0} T_m \right)$$

$$= -\left(C_{Bvar_{-9}}^{E0} \frac{1}{2} I \right)^{-1} \frac{1}{2} \underline{g}_{avg}^{E0} T_m = -\left(C_{Bvar_{-9}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m$$

For $m = -8, -6, -4, -2, 0$: (G-13)

$$\Delta \underline{\upsilon}_{var_m}^{Bvar} = \left(C_{Bvar_{m-1}}^{E0} G_{Rvar_m} \right)^{-1} \left[C_{Bvar_{m-1}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} \right) - \underline{V}_{var_{m-1}}^{E0} + \underline{V}_{ref_0}^{E0} - \frac{1}{2} \underline{g}_{avg}^{E0} T_m \right]$$

$$= 2 \left[I - \frac{1}{3} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] \left(C_{Bvar_{m-1}}^{E0} \right)^{-1} \left[C_{Bvar_{m-1}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} \right) - \underline{V}_{ref_0}^{E0} + \underline{V}_{ref_0}^{E0} - \frac{1}{2} \underline{g}_{avg}^{E0} T_m \right]$$

$$\approx 2 \, \Delta \underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} - \left[I - \frac{1}{3} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] \left(C_{Bvar_{m-1}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m$$

For $m = -7, -5, -3, -1$:

$$
\begin{aligned}
\Delta \underline{\upsilon}_{var_m}^{Bvar} &= \left(C_{Bvar_{m-1}}^{E0}\, G_{Rvar_m}\right)^{-1}\left[C_{Bvar_{m-1}}^{E0}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m}\right) - \underline{V}_{var_{m-1}}^{E0} + \underline{V}_{ref_0}^{E0} - \frac{1}{2}\underline{g}_{avg}^{E0}\, T_m\right] \\
&= 2\left[I - \frac{1}{3}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times\right)\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\left\{\begin{array}{l} C_{Bvar_{m-1}}^{E0}\,\Delta\underline{\alpha}_{var}^{Bvar} \times \dfrac{\underline{l}^{Bref}}{T_m} - 2\, C_{Bvar_{m-2}}^{E0}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times \dfrac{\underline{l}^{Bref}}{T_m}\right) \\ \qquad + \dfrac{1}{6}\left(C_{Bvar_{m-2}}^{E0}\,\Delta\underline{\alpha}_{var}^{Bvar}\right) \times \underline{g}^{E0}\, T_m - \dfrac{1}{2}\underline{g}_{avg}^{E0}\, T_m \end{array}\right\} \\
&\approx 2\,\Delta\underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} - 4\left(C_{Bvar_{m-1}}^{E0}\right)^{-1} C_{Bvar_{m-2}}^{E0}\,\Delta\underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} \qquad \text{(G-14)} \\
&+ \frac{1}{3}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\left[\left(C_{Bvar_{m-2}}^{E0}\,\Delta\underline{\alpha}_{var}^{Bvar}\right) \times \underline{g}_{avg}^{E0}\, T_m\right] - \left[I - \frac{1}{3}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times\right)\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m \\
&\approx -2\,\Delta\underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} + \frac{1}{3}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\left[C_{Bvar_{m-1}}^{E0}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times\right)\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m\right] \\
&\quad + \frac{1}{3}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times\right)\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m - \left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m \\
&= -2\,\Delta\underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} + \frac{2}{3}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times\right)\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m - \left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m \\
&= -2\,\Delta\underline{\alpha}_{var}^{Bvar} \times \frac{\underline{l}^{Bref}}{T_m} - \left[I - \frac{2}{3}\left(\Delta\underline{\alpha}_{var}^{Bvar} \times\right)\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m
\end{aligned}
$$

For $m = 1, 3, 5, \cdots$:

$$\Delta\underline{\upsilon}_{var_m}^{Bvar} = \left(C_{Bvar_{m-1}}^{E0}\, G_{Rvar_m}\right)^{-1}\left(C_{Bvar_{m-1}}^{E0}\,\Delta\underline{\alpha}'^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} - \underline{V}_{var_{m-1}}^{E0} + \underline{V}_{ref_0}^{E0} - \frac{1}{2}\underline{g}_{avg}^{E0}\, T_m\right)$$

$$= 2\left[I - \frac{1}{3}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\left\{\begin{array}{c} C_{Bvar_{m-1}}^{E0}\,\Delta\underline{\alpha}'^{Bvar}_{var}\times\dfrac{\underline{l}^{Bref}}{T_m} - 2\, C_{Bvar_{m-2}}^{E0}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\dfrac{\underline{l}^{Bref}}{T_m}\right) \\ + \dfrac{1}{6}\left(C_{Bvar_{m-2}}^{E0}\,\Delta\underline{\alpha}_{var}^{Bvar}\right)\times\underline{g}^{E0}\, T_m - \dfrac{1}{2}\underline{g}_{avg}^{E0}\, T_m \end{array}\right\}$$

$$\approx 2\,\Delta\underline{\alpha}'^{Bvar}_{var}\times\frac{\underline{l}^{Bref}}{T_m} - 4\left(C_{Bvar_{m-1}}^{E0}\right)^{-1} C_{Bvar_{m-2}}^{E0}\,\Delta\underline{\alpha}_{var}^{Bvar}\times\frac{\underline{l}^{Bref}}{T_m} \tag{G-15}$$

$$+\frac{1}{3}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\left[\left(C_{Bvar_{m-2}}^{E0}\,\Delta\underline{\alpha}_{var}^{Bvar}\right)\times\underline{g}_{avg}^{E0}\, T_m\right] - \left[I - \frac{1}{3}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

$$\approx 2\left(\Delta\underline{\alpha}'^{Bvar}_{var} - 2\,\Delta\underline{\alpha}_{var}^{Bvar}\right)\times\frac{\underline{l}^{Bref}}{T_m} + \frac{1}{3}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\left[C_{Bvar_{m-1}}^{E0}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m\right]$$

$$+\frac{1}{3}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m - \left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

$$= 2\left(\Delta\underline{\alpha}'^{Bvar}_{var} - 2\,\Delta\underline{\alpha}_{var}^{Bvar}\right)\times\frac{\underline{l}^{Bref}}{T_m} + \frac{1}{3}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

$$-\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

$$= 2\left(\Delta\underline{\alpha}'^{Bvar}_{var} - 2\,\Delta\underline{\alpha}_{var}^{Bvar}\right)\times\frac{\underline{l}^{Bref}}{T_m} - \left\{I - \frac{1}{3}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\right\}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

For $m = 2, 4, 6, \cdots$:

$$\Delta\underline{\upsilon}_{var_m}^{Bvar} = \left(C_{Bvar_{m-1}}^{E0}\,G_{Rvar_m}\right)^{-1}\left(C_{Bvar_{m-1}}^{E0}\,\Delta\underline{\alpha}'^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} - \underline{V}_{var_{m-1}}^{E0} + \underline{V}_{ref_0}^{E0} - \frac{1}{2}\,\underline{g}_{avg}^{E0}\,T_m\right)$$

$$= 2\left[I - \frac{1}{3}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\left\{\begin{array}{c} C_{Bvar_{m-1}}^{E0}\,\Delta\underline{\alpha}'^{Bvar}_{var} \times \dfrac{\underline{l}^{Bref}}{T_m} \\ -2\,C_{Bvar_{m-2}}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\dfrac{\underline{l}^{Bref}}{T_m}\right] \\ +\dfrac{1}{6}\left[C_{Bvar_{m-2}}^{E0}\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\right]\times\underline{g}_{avg}^{E0}\,T_m - \dfrac{1}{2}\,\underline{g}_{avg}^{E0}\,T_m \end{array}\right\}$$

$$\approx 2\,\Delta\underline{\alpha}'^{Bvar}_{var} \times \frac{\underline{l}^{Bref}}{T_m} - 4\left(C_{Bvar_{m-1}}^{E0}\right)^{-1} C_{Bvar_{m-2}}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\frac{\underline{l}^{Bref}}{T_m}\right]$$
$$+\frac{1}{3}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\left\{\left[C_{Bvar_{m-2}}^{E0}\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\right]\times\underline{g}_{avg}^{E0}\,T_m\right\} \qquad \text{(G-16)}$$
$$-\left[I - \frac{1}{3}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\,T_m$$

$$\approx -2\left(\Delta\underline{\alpha}'^{Bvar}_{var} - 2\,\Delta\underline{\alpha}_{var}^{Bvar}\right)\times\frac{\underline{l}^{Bref}}{T_m}$$
$$+\frac{1}{3}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\left\{C_{Bvar_{m-1}}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\,T_m\right\}$$
$$+\frac{1}{3}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\,T_m - \left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\,T_m$$

$$= -2\left(\Delta\underline{\alpha}'^{Bvar}_{var} - 2\,\Delta\underline{\alpha}_{var}^{Bvar}\right)\times\frac{\underline{l}^{Bref}}{T_m} + \frac{1}{3}\left[\left(2\,\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\,T_m$$
$$-\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\,T_m$$

$$= -2\left(\Delta\underline{\alpha}'^{Bvar}_{var} - 2\,\Delta\underline{\alpha}_{var}^{Bvar}\right)\times\frac{\underline{l}^{Bref}}{T_m} - \left\{I - \frac{1}{3}\left[\left(2\,\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\right\}\left(C_{Bvar_{m-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\,T_m$$

The combined (G-13) – (G-16) result for $\Delta\underline{\upsilon}_{var_m}^{Bvar}$ specific force is then as shown in (G-7).

APPENDIX H

ANALYTICAL DETAIL FOR METHOD 2 TEST EXAMPLE SOLUTION

This appendix provides analytical detail leading to the Method 2 test example solution of Eqs. (31) for specific force and resulting velocity, position thereof.

SPECIFIC FORCE

Under test example conditions, Method 2 performance is demonstrated with gravity approximated to be constant $\underline{g}^{E0}$ as in (38). Then (31) simplifies to

$$
\begin{aligned}
A_{2n-1} &\equiv C_{Bvar_{2n-1}}^{E0} G_{Rvar_{2n}} \left(C_{Bvar_{2n-1}}^{E0} G_{Vvar_{2n}} \right)^{-1} \\
B_{den_{2n-1}} &\equiv \left[C_{Bvar_{2n-2}}^{E0} \left(G_{Rvar_{2n-1}} + G_{Vvar_{2n-1}} \right) - A_{2n-1} C_{Bvar_{2n-2}}^{E0} G_{Vvar_{2n-1}} \right] T_m \\
B_{num_{2n-1}} &\equiv \underline{R}_{var_{2n}}^{E0} - \underline{R}_{var_{2n-2}}^{E0} - 2 \underline{V}_{var_{2n-2}}^{E0} T_m - A_{2n-1} \left(\underline{V}_{var_{2n}}^{E0} - \underline{V}_{var_{2n-2}}^{E0} \right) T_m \\
&\quad + 2 \left(A_{2n-1} - I \right) \underline{g}_{avg}^{E0} T_m^2 \\
\Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar} &= B_{den_{2n-1}}^{-1} B_{num_{2n-1}} \\
A_{2n} &\equiv C_{Bvar_{2n-2}}^{E0} \left(G_{Rvar_{2n-1}} + G_{Vvar_{2n-1}} \right) \left(C_{Bvar_{2n-2}}^{E0} G_{Vvar_{2n-1}} \right)^{-1} \\
B_{den_{2n}} &\equiv \left(C_{Bvar_{2n-1}}^{E0} G_{Rvar_{2n}} - A_{2n} C_{Bvar_{2n-1}}^{E0} G_{Vvar_{2n}} \right) T_m \\
B_{num_{2n}} &\equiv \underline{R}_{var_{2n}}^{E0} - \underline{R}_{var_{2n-2}}^{E0} - 2 \underline{V}_{var_{2n-2}}^{E0} T_m - A_{2n} \left(\underline{V}_{var_{2n}}^{E0} - \underline{V}_{var_{2n-2}}^{E0} \right) T_m \\
&\quad + 2 \left(A_{2n} - I \right) \underline{g}_{avg}^{E0} T_m^2 \\
\Delta \underline{\upsilon}_{var_{2n}}^{Bvar} &= B_{den_{2n}}^{-1} B_{num_{2n}}
\end{aligned}
\tag{H-1}
$$

The development sequence for deriving $\Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar}$ and $\Delta \underline{\upsilon}_{var_{2n}}^{Bvar}$ in (H-1) for the test example is, based on (32) and (33), first finding A_{2n-1}, A_{2n}, then $B_{den_{2n-1}}, B_{den_{2n}}$, then $\underline{R}_{var_{2n}}^{E0}$, $\underline{V}_{var_{2n}}^{E0}$, then $B_{num_{2n}}$, $B_{num_{2n-1}}$, and from those, $\Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar}$ and $\Delta \underline{\upsilon}_{var_{2n}}^{Bvar}$.

Finding A_{2n-1} And A_{2n}

To derive A_{2n-1}, A_{2n}, first expand their equations in (H-1):

$$A_{2n-1} \equiv C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} \left(C^{E0}_{Bvar_{2n-1}} G_{Vvar_{2n}} \right)^{-1}$$

$$= C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} \left(G_{Vvar_{2n}} \right)^{-1} \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1}$$

$$A_{2n} \equiv C^{E0}_{Bvar_{2n-2}} \left(G_{Rvar_{2n-1}} + G_{Vvar_{2n-1}} \right) \left(C^{E0}_{Bvar_{2n-2}} G_{Vvar_{2n-1}} \right)^{-1} \qquad \text{(H-2)}$$

$$= C^{E0}_{Bvar_{2n-2}} G_{Rvar_{2n-1}} \left(C^{E0}_{Bvar_{2n-2}} G_{Vvar_{2n-1}} \right)^{-1} + I$$

$$= C^{E0}_{Bvar_{2n-2}} G_{Rvar_{2n-1}} \left(G_{Vvar_{2n-1}} \right)^{-1} \left(C^{E0}_{Bvar_{2n-2}} \right)^{-1} + I$$

To obtain $G_{Rvar_{2n}} \left(G_{Vvar_{2n}} \right)^{-1}$ in (H-2), apply (33) for $G_{Vvar_{2n}}$ and $G_{Rvar_{2n}}$, yielding to first order accuracy:

$$\text{For } n \leq -5: \quad G_{Rvar_{2n}} \left(G_{Vvar_{2n}} \right)^{-1} = \frac{1}{2} I$$

$$\text{For } 0 \geq n \geq -4:$$

$$G_{Rvar_{2n}} \left(G_{Vvar_{2n}} \right)^{-1} \approx \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right]^{-1} \qquad \text{(H-3)}$$

$$= \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] \left[I - \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] \approx \frac{1}{2} \left[I - \frac{1}{6} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right]$$

$$\text{For } n > 0: \quad G_{Rvar_{2n}} \left(G_{Vvar_{2n}} \right)^{-1} \approx \frac{1}{2} \left[I - \frac{1}{6} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right]$$

Substituting (H-3) in (H-2) and dropping higher order terms then obtains A_{2n-1}, A_{2n} :

$$\text{For } n \le -5:$$

$$A_{2n-1} = C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} \left(G_{Vvar_{2n}}\right)^{-1} \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}$$

$$= C^{E0}_{Bvar_{2n-1}} \frac{1}{2} I \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1} = \frac{1}{2} I$$

$$A_{2n} = C^{E0}_{Bvar_{2n-2}} G_{Rvar_{2n-1}} \left(G_{Vvar_{2n-1}}\right)^{-1} \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} + I$$

$$= C^{E0}_{Bvar_{2n-2}} \frac{1}{2} I \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} + I = \frac{3}{2} I$$

$$\text{For } n = -4:$$

$$A_{2n-1} = C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} \left(G_{Vvar_{2n}}\right)^{-1} \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}$$

$$= C^{E0}_{Bvar_{2n-1}} \frac{1}{2} \left[I - \frac{1}{6} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}$$

$$= \frac{1}{2} \left[I - \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1} \right]$$

$$A_{2n} = C^{E0}_{Bvar_{2n-2}} G_{Rvar_{2n-1}} \left(G_{Vvar_{2n-1}}\right)^{-1} \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} + I = \frac{3}{2} I \qquad \text{(H-4)}$$

$$\text{For } 0 \ge n \ge -3:$$

$$A_{2n-1} = \frac{1}{2} \left[I - \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1} \right]$$

$$A_{2n} = C^{E0}_{Bvar_{2n-2}} G_{Rvar_{2n-1}} \left(G_{Vvar_{2n-1}}\right)^{-1} \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} + I$$

$$= C^{E0}_{Bvar_{2n-2}} \frac{1}{2} \left[I - \frac{1}{6} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} + I$$

$$= \frac{3}{2} \left[I - \frac{1}{18} C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} \right]$$

$$\text{For } n > 0:$$

$$A_{2n-1} = \frac{1}{2} \left[I - \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1} \right]$$

$$A_{2n} = \frac{3}{2} \left[I - \frac{1}{18} C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} \right]$$

Summary of (H-4) Results:

$$\text{For } n \le -5: \quad A_{2n-1} = \frac{1}{2} I \qquad A_{2n} = \frac{3}{2} I$$

$$\text{For } n = -4:$$

$$A_{2n-1} = \frac{1}{2}\left[I - \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \right] \qquad A_{2n} = \frac{3}{2} I$$

$$\text{For } 0 \ge n \ge -3:$$

$$A_{2n-1} = \frac{1}{2}\left[I - \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \right] \qquad \text{(H-5)}$$

$$A_{2n} = \frac{3}{2}\left[I - \frac{1}{18} C^{E0}_{Bvar_{2n-2}} \left(\Delta\underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-2}} \right)^{-1} \right]$$

$$\text{For } n > 0:$$

$$A_{2n-1} = \frac{1}{2}\left[I - \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \right]$$

$$A_{2n} = \frac{3}{2}\left[I - \frac{1}{18} C^{E0}_{Bvar_{2n-2}} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-2}} \right)^{-1} \right]$$

Finding $B_{den_{2n-1}}$ And $B_{den_{2n}}$

To determine $B_{den_{2n-1}}$ and $B_{den_{2n}}$, substitute (33) and (H-4) into the $B_{den_{2n-1}}$, $B_{den_{2n}}$ expressions in (H-2), and drop higher order terms:

$$\text{For } n < -4:$$

$$B_{den_{2n-1}} = \left[C^{E0}_{Bvar_{2n-2}} \left(G_{Rvar_{2n-1}} + G_{Vvar_{2n-1}} \right) - A_{2n-1} C^{E0}_{Bvar_{2n-2}} G_{Vvar_{2n-1}} \right] T_m$$

$$= \left[C^{E0}_{Bvar_{2n-2}} \left(\frac{1}{2} I + I \right) - \frac{1}{2} I \, C^{E0}_{Bvar_{2n-2}} \, I \right] T_m = C^{E0}_{Bvar_{2n-2}} T_m = C^{E0}_{Bvar_{-9}} T_m \qquad \text{(H-6)}$$

$$B_{den_{2n}} = \left(C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} - A_{2n} \, C^{E0}_{Bvar_{2n-1}} G_{Vvar_{2n}} \right) T_m$$

$$= \left(C^{E0}_{Bvar_{2n-1}} \frac{1}{2} I - \frac{3}{2} I \, C^{E0}_{Bvar_{2n-1}} \, I \right) T_m = - C^{E0}_{Bvar_{2n-1}} T_m = - C^{E0}_{Bvar_{-9}} T_m$$

For $n = -4$:

$$B_{den_{2n-1}} = \left[C^{E0}_{Bvar_{2n-2}} \left(G_{Rvar_{2n-1}} + G_{Vvar_{2n-1}} \right) - A_{2n-1} C^{E0}_{Bvar_{2n-2}} G_{Vvar_{2n-1}} \right] T_m$$

$$= \left\{ C^{E0}_{Bvar_{2n-2}} \left(\frac{1}{2} I + I \right) - \frac{1}{2} \left[I - \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \right] C^{E0}_{Bvar_{2n-2}} I \right\} T_m$$

$$= C^{E0}_{Bvar_{2n-2}} \left[I + \frac{1}{12} \left(C^{E0}_{Bvar_{2n-2}} \right)^{-1} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} C^{E0}_{Bvar_{2n-2}} \right] T_m$$

$$= C^{E0}_{Bvar_{-9}} \left[I + \frac{1}{12} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] T_m \qquad \text{(H-7)}$$

$$B_{den_{2n}} = \left(C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} - A_{2n} C^{E0}_{Bvar_{2n-1}} G_{Vvar_{2n}} \right) T_m$$

$$= \left\{ C^{E0}_{Bvar_{2n-1}} \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] - \frac{3}{2} I \, C^{E0}_{Bvar_{2n-1}} \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] \right\} T_m$$

$$= - C^{E0}_{Bvar_{-9}} \left[I + \frac{7}{12} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \right] T_m$$

For $0 \ge n > -4$:

$$B_{den_{2n-1}} = \left[C_{Bvar_{2n-2}}^{E0} \left(G_{Rvar_{2n-1}} + G_{Vvar_{2n-1}} \right) - A_{2n-1} C_{Bvar_{2n-2}}^{E0} G_{Vvar_{2n-1}} \right] T_m$$

$$= \left\langle \begin{array}{c} C_{Bvar_{2n-2}}^{E0} \left\{ \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] + \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] \right\} \\ - \frac{1}{2} \left[I - \frac{1}{6} C_{Bvar_{2n-1}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \left(C_{Bvar_{2n-1}}^{E0} \right)^{-1} \right] C_{Bvar_{2n-2}}^{E0} \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] \end{array} \right\rangle T_m$$

$$= \left\{ \begin{array}{c} C_{Bvar_{2n-2}}^{E0} \left[I + \frac{5}{12} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] \\ + \frac{1}{12} C_{Bvar_{2n-1}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \left(C_{Bvar_{2n-1}}^{E0} \right)^{-1} C_{Bvar_{2n-2}}^{E0} \end{array} \right\} T_m \qquad \text{(H-8)}$$

$$= C_{Bvar_{2n-2}}^{E0} \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] T_m$$

$$B_{den_{2n}} = \left(C_{Bvar_{2n-1}}^{E0} G_{Rvar_{2n}} - A_{2n} C_{Bvar_{2n-1}}^{E0} G_{Vvar_{2n}} \right) T_m$$

$$= \left\{ \begin{array}{c} C_{Bvar_{2n-1}}^{E0} \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] \\ - \frac{3}{2} \left[I - \frac{1}{18} C_{Bvar_{2n-2}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \left(C_{Bvar_{2n-2}}^{E0} \right)^{-1} \right] C_{Bvar_{2n-1}}^{E0} \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] \end{array} \right\} T_m$$

$$= \left\{ \begin{array}{c} - C_{Bvar_{2n-1}}^{E0} + \frac{1}{6} C_{Bvar_{2n-1}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) - \frac{3}{4} C_{Bvar_{2n-1}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \\ + \frac{1}{12} C_{Bvar_{2n-2}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \end{array} \right\} T_m$$

$$= - C_{Bvar_{2n-1}}^{E0} \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] T_m$$

For $n > 0$:

$$B_{den_{2n-1}} = \left[C^{E0}_{Bvar_{2n-2}} \left(G_{Rvar_{2n-1}} + G_{Vvar_{2n-1}} \right) - A_{2n-1} C^{E0}_{Bvar_{2n-2}} G_{Vvar_{2n-1}} \right] T_m$$

$$= \begin{bmatrix} C^{E0}_{Bvar_{2n-2}} \left\{ \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] + \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] \right\} \\ -\frac{1}{2} \left[I - \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \right] C^{E0}_{Bvar_{2n-2}} \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] \end{bmatrix} T_m$$

$$= \begin{bmatrix} C^{E0}_{Bvar_{2n-2}} + \frac{2}{3} C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) - \frac{1}{4} C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \\ + \frac{1}{12} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} C^{E0}_{Bvar_{2n-2}} \end{bmatrix} T_m$$

$$= C^{E0}_{Bvar_{2n-2}} \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] T_m \qquad \text{(H-9)}$$

$$B_{den_{2n}} = \left(C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} - A_{2n} C^{E0}_{Bvar_{2n-1}} G_{Vvar_{2n}} \right) T_m$$

$$= \begin{Bmatrix} C^{E0}_{Bvar_{2n-1}} \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] \\ -\frac{3}{2} \left[I - \frac{1}{18} C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-2}} \right)^{-1} \right] C^{E0}_{Bvar_{2n-1}} \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] \end{Bmatrix} T_m$$

$$= \begin{Bmatrix} -C^{E0}_{Bvar_{2n-1}} + \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) - \frac{3}{4} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \\ + \frac{1}{12} C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \end{Bmatrix} T_m$$

$$= -C^{E0}_{Bvar_{2n-1}} \left[I + \frac{1}{2} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] T_m$$

Summary of (H-6) to (H-9) Results:

For $n < -4$:

$$B_{den_{2n-1}} = C^{E0}_{Bvar_{-9}}\,T_m \qquad B_{den_{2n}} = -\,C^{E0}_{Bvar_{-9}}\,T_m$$

For $n = -4$:

$$B_{den_{2n-1}} = C^{E0}_{Bvar_{-9}}\left[I + \frac{1}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]T_m \qquad B_{den_{2n}} = -\,C^{E0}_{Bvar_{-9}}\left[I + \frac{7}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]T_m$$

For $0 \geq n > -4$: (H-10)

$$B_{den_{2n-1}} = C^{E0}_{Bvar_{2n-2}}\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]T_m \qquad B_{den_{2n}} = -\,C^{E0}_{Bvar_{2n-1}}\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]T_m$$

For $n > 0$:

$$B_{den_{2n-1}} = C^{E0}_{Bvar_{2n-2}}\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]T_m \qquad B_{den_{2n}} = -\,C^{E0}_{Bvar_{2n-1}}\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]T_m$$

Finding The $\underline{R}^{E0}_{var_{2n}}$ And $\underline{V}^{E0}_{var_{2n}}$ Terms

Eq. (37) for $\underline{R}^{E0}_{var_m}$ with $m = 2n$ is used as the basis for evaluating the $\underline{R}^{E0}_{var_{2n}}$, $\underline{V}^{E0}_{var_{2n}}$ terms in (H-1) under the example conditions.

$$\underline{R}^{E0}_{var_{2n}} = 2n\,\underline{V}^{E0}_{ref_0}\,T_m + C^{E0}_{Bvar_{2n}}\,\underline{l}^{Bref}$$

$$\underline{V}^{E0}_{var_{2n}} = \underline{V}^{E0}_{var_m} = \frac{\underline{R}^{E0}_{var_{m+1}} - \underline{R}^{E0}_{var_{m-1}}}{2\,T_m}$$

$$= \frac{(m+1)\,\underline{V}^{E0}_{ref_0}\,T_m + C^{E0}_{Bvar_{m+1}}\,\underline{l}^{Bref} - (m-1)\,\underline{V}^{E0}_{ref_0}\,T_m - C^{E0}_{Bvar_{m-1}}\,\underline{l}^{Bref}}{2\,T_m} \qquad \text{(H-11)}$$

$$= \underline{V}^{E0}_{ref_0} + \frac{\left(C^{E0}_{Bvar_{2n+1}} - C^{E0}_{Bvar_{2n-1}}\right)\underline{l}^{Bref}}{2\,T_m}$$

From (H-11) with (32) for $C^{E0}_{Bvar_m}$, the $\underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}}$ position change in (H-1) becomes:

$$\text{For } n < -4: \quad \underline{R}^{E0}_{var\,2n} - \underline{R}^{E0}_{var\,2n-2} = 2\,\underline{V}^{E0}_{ref_0}\ T_m$$

For $n = -4$:

$$C^{E0}_{Bvar\,2n} = C^{E0}_{Bvar\,-8} = C^{E0}_{Bvar\,-9}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right] \quad C^{E0}_{Bvar\,n-2} = C^{E0}_{Bvar\,-9}$$

$$C^{E0}_{Bvar\,n} - C^{E0}_{Bvar\,n-2} = C^{E0}_{Bvar\,-8} - C^{E0}_{Bvar\,-9} = C^{E0}_{Bvar\,-9}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$\underline{R}^{E0}_{var\,2n} - \underline{R}^{E0}_{var\,2n-2} = \underline{R}^{E0}_{var\,-8} - \underline{R}^{E0}_{var\,-10} = 2\,\underline{V}^{E0}_{ref_0}\ T_m + \left(C^{E0}_{Bvar\,-8} - C^{E0}_{Bvar\,-9} \right) \underline{l}^{Bref}$$

$$= 2\,\underline{V}^{E0}_{ref_0}\ T_m + C^{E0}_{Bvar\,-9}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right] \underline{l}^{Bref}$$

For $0 \geq n > -4$:

$$C^{E0}_{Bvar\,2n} = C^{E0}_{Bvar\,2n-1}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$C^{E0}_{Bvar\,2n-1} = C^{E0}_{Bvar\,2n-2}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right] \qquad \text{(H-12)}$$

$$C^{E0}_{Bvar\,2n} = C^{E0}_{Bvar\,2n-2}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar\,2n-2}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar\,2n-2}\left[I + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 + \cdots \right]$$

$$C^{E0}_{Bvar\,2n} - C^{E0}_{Bvar\,2n-2} = C^{E0}_{Bvar\,2n-2}\left[I + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 + \cdots - I \right]$$

$$\approx 2\,C^{E0}_{Bvar\,2n-2}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$\underline{R}^{E0}_{var\,2n} - \underline{R}^{E0}_{var\,2n-2} \approx 2\,\underline{V}^{E0}_{ref_0}\ T_m + 2\,C^{E0}_{Bvar\,2n-2}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right] \underline{l}^{Bref}$$

For $n > 0$:

$$\underline{R}^{E0}_{var\,2n} - \underline{R}^{E0}_{var\,2n-2} \approx 2\,\underline{V}^{E0}_{ref_0}\ T_m + 2\,C^{E0}_{Bvar\,2n-2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right] \underline{l}^{Bref}$$

Summary of (H-12) Results:

$$\text{For } n < -4: \quad \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} = 2\,\underline{V}^{E0}_{ref_0}\;T_m$$

$$\text{For } n = -4:$$

$$\underline{R}^{E0}_{var_{-8}} - \underline{R}^{E0}_{var_{-10}} = 2\,\underline{V}^{E0}_{ref_0}\;T_m + C^{E0}_{Bvar_{-9}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref}$$

$$\text{For } 0 \geq n > -4: \qquad\qquad \text{(H-13)}$$

$$\underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} \approx 2\,\underline{V}^{E0}_{ref_0}\;T_m + 2\,C^{E0}_{Bvar_{2n-2}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref}$$

$$\text{For } n > 0:$$

$$\underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} \approx 2\,\underline{V}^{E0}_{ref_0}\;T_m + 2\,C^{E0}_{Bvar_{2n-2}}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref}$$

The $\underline{V}^{E0}_{var_{2n-2}}$ and $\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}}$ velocity terms in (H-1) are obtained from (H-11) with (32) for $C^{E0}_{Bvar_m}$. First, $\underline{V}^{E0}_{var_{2n}}$ is obtained. Then what follows is modified to get $\underline{V}^{E0}_{var_{2n-2}}$ and $\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}}$.

For $n < -4$:

$$\underline{R}^{E0}_{var\,2n+1} - \underline{R}^{E0}_{var\,2n-1} = 2\,\underline{V}^{E0}_{ref_0}\ T_m + \left(C^{E0}_{Bvar\,2n+1} - C^{E0}_{Bvar\,2n-1}\right)\underline{l}^{Bref}$$

$$= 2\,\underline{V}^{E0}_{ref_0}\ T_m + \left(C^{E0}_{Bvar_{-9}} - C^{E0}_{Bvar_{-9}}\right)\underline{l}^{Bref} = 2\,\underline{V}^{E0}_{ref_0}\ T_m$$

$$\underline{V}^{E0}_{var_{-10}} = \frac{\underline{R}^{E0}_{var\,2n+1} - \underline{R}^{E0}_{var\,2n-1}}{2\,T_m} = \underline{V}^{E0}_{ref_0}$$

For $n = -4$:

$$C^{E0}_{Bvar_{-8}} = C^{E0}_{Bvar_{-9}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]$$

$$C^{E0}_{Bvar_{-7}} = C^{E0}_{Bvar_{-8}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]$$

$$= C^{E0}_{Bvar_{-9}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]$$

$$= C^{E0}_{Bvar_{-9}}\left[I + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 + \cdots\right] \qquad \text{(H-14)}$$

$$C^{E0}_{Bvar_{-7}} - C^{E0}_{Bvar_{-9}} = C^{E0}_{Bvar_{-9}}\left[I + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 + \cdots - I\right]$$

$$\approx 2\,C^{E0}_{Bvar_{-9}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]$$

$$\underline{R}^{E0}_{var_{-7}} - \underline{R}^{E0}_{var_{-9}} = 2\,\underline{V}^{E0}_{ref_0}\ T_m + \left(C^{E0}_{Bvar_{-7}} - C^{E0}_{Bvar_{-9}}\right)\underline{l}^{Bref}$$

$$= 2\,\underline{V}^{E0}_{ref_0}\ T_m + 2\,C^{E0}_{Bvar_{-9}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref}$$

$$\underline{V}^{E0}_{var_{-8}} = \frac{\underline{R}^{E0}_{var_{-7}} - \underline{R}^{E0}_{var_{-9}}}{2\,T_m} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-9}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

For $n = -3$:

$$C_{Bvar_{-7}}^{E0} = C_{Bvar_{-8}}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$C_{Bvar_{-6}}^{E0} = C_{Bvar_{-7}}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$C_{Bvar_{-5}}^{E0} = C_{Bvar_{-6}}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$= C_{Bvar_{-7}}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$= C_{Bvar_{-8}}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]\left[I + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 + \cdots \right]$$

$$= C_{Bvar_{-8}}^{E0}\left[I + 3\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{9}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \cdots \right] \qquad \text{(H-15)}$$

$$C_{Bvar_{-5}}^{E0} - C_{Bvar_{-7}}^{E0} \approx 2\, C_{Bvar_{-8}}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$\underline{R}_{var_{-5}}^{E0} - \underline{R}_{var_{-7}}^{E0} = 2\,\underline{V}_{ref_0}^{E0}\; T_m + \left(C_{Bvar_{-5}}^{E0} - C_{Bvar_{-7}}^{E0} \right) \underline{l}^{Bref}$$

$$= 2\,\underline{V}_{ref_0}^{E0}\; T_m + 2\, C_{Bvar_{-8}}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right] \underline{l}^{Bref}$$

$$\underline{V}_{var_{-6}}^{E0} = \frac{\underline{R}_{var_{-5}}^{E0} - \underline{R}_{var_{-7}}^{E0}}{2\, T_m} = \underline{V}_{ref_0}^{E0} + C_{Bvar_{-8}}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right] \frac{\underline{l}^{Bref}}{T_m}$$

For $n = -2$:

$$C^{E0}_{Bvar-5} = C^{E0}_{Bvar-6}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$C^{E0}_{Bvar-4} = C^{E0}_{Bvar-5}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$C^{E0}_{Bvar-3} = C^{E0}_{Bvar-4}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar-5}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar-6}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]\left[I + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 + \cdots \right]$$

$$= C^{E0}_{Bvar-6}\left[I + 3\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{9}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \cdots \right] \qquad \text{(H-16)}$$

$$C^{E0}_{Bvar-3} - C^{E0}_{Bvar-5} \approx 2\, C^{E0}_{Bvar-6}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$\underline{R}^{E0}_{var-3} - \underline{R}^{E0}_{var-5} = 2\,\underline{V}^{E0}_{ref_0}\, T_m + 2\, C^{E0}_{Bvar-6}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]\underline{l}^{Bref}$$

$$\underline{V}^{E0}_{var-4} = \frac{\underline{R}^{E0}_{var-3} - \underline{R}^{E0}_{var-5}}{2\, T_m} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar-6}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]\frac{\underline{l}^{Bref}}{T_m}$$

For $n = -1$:

$$C_{Bvar-3}^{E0} = C_{Bvar-4}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$C_{Bvar-2}^{E0} = C_{Bvar-3}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$C_{Bvar-1}^{E0} = C_{Bvar-2}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$= C_{Bvar-3}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$= C_{Bvar-4}^{E0}\left[I + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]\left[I + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 + \cdots \right]$$

$$= C_{Bvar-4}^{E0}\left[I + 3\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \frac{9}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \cdots \right] \qquad \text{(H-17)}$$

$$C_{Bvar-1}^{E0} - C_{Bvar-3}^{E0} \approx 2\, C_{Bvar-4}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]$$

$$\underline{R}_{var-1}^{E0} - \underline{R}_{var-3}^{E0} = 2\, \underline{V}_{ref_0}^{E0}\, T_m + 2\, C_{Bvar-4}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]\underline{l}^{Bref}$$

$$\underline{V}_{var-2}^{E0} = \frac{\underline{R}_{var-1}^{E0} - \underline{R}_{var-3}^{E0}}{2\, T_m} = \underline{V}_{ref_0}^{E0} + C_{Bvar-4}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 \right]\frac{\underline{l}^{Bref}}{T_m}$$

For $n = 0$:

$$C^{E0}_{Bvar_{-1}} = C^{E0}_{Bvar_{-2}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$C^{E0}_{Bvar_0} = C^{E0}_{Bvar_{-1}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$C^{E0}_{Bvar_1} = C^{E0}_{Bvar_0}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar_{-1}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar_{-1}}\left\{ I + \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] + \frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 \right\}$$

$$\approx C^{E0}_{Bvar_{-2}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]\left\{ \begin{array}{l} I + \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ + \frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 \end{array} \right\}$$

$$= C^{E0}_{Bvar_{-2}}\left\{ I + \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] + \frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 \right\} \qquad \text{(H-18)}$$

$$C^{E0}_{Bvar_1} - C^{E0}_{Bvar_{-1}} \approx C^{E0}_{Bvar_{-2}}\left\langle \begin{array}{c} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ + \frac{1}{2}\left\{\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right\} \end{array} \right\rangle$$

$$\underline{R}^{E0}_{var_1} - \underline{R}^{E0}_{var_{-1}} = 2\,\underline{V}^{E0}_{ref_0}\,T_m + \left(C^{E0}_{Bvar_1} - C^{E0}_{Bvar_{-1}}\right)\underline{l}^{Bref}$$

$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + C^{E0}_{Bvar_{-2}}\left\langle \begin{array}{c} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ + \frac{1}{2}\left\{\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right\} \end{array} \right\rangle \underline{l}^{Bref}$$

$$\underline{V}^{E0}_{var_0} = \frac{\underline{R}^{E0}_{var_1} - \underline{R}^{E0}_{var_{-1}}}{2\,T_m}$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_{-2}}\left\langle \begin{array}{c} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ + \frac{1}{2}\left\{\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right\} \end{array} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

For $n=1$:

$$C^{E0}_{Bvar1} = C^{E0}_{Bvar0}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$C^{E0}_{Bvar2} = C^{E0}_{Bvar1}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$C^{E0}_{Bvar3} = C^{E0}_{Bvar2}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar1}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar1}\left[I + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right] \qquad \text{(H-19)}$$

$$= C^{E0}_{Bvar0}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]\left[I + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar0}\left[I + 3\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{9}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$C^{E0}_{Bvar3} - C^{E0}_{Bvar1} \approx 2\, C^{E0}_{Bvar0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$\underline{R}^{E0}_{var3} - \underline{R}^{E0}_{var1} = 2\,\underline{V}^{E0}_{ref_0}\, T_m + \left(C^{E0}_{Bvar3} - C^{E0}_{Bvar1} \right) \underline{l}^{Bref}$$

$$= 2\,\underline{V}^{E0}_{ref_0}\, T_m + 2\, C^{E0}_{Bvar0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right] \underline{l}^{Bref}$$

$$\underline{V}^{E0}_{var2} = \frac{\underline{R}^{E0}_{var3} - \underline{R}^{E0}_{var1}}{2\, T_m} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right] \frac{\underline{l}^{Bref}}{T_m}$$

For $n = 2$:

$$C_{Bvar3}^{E0} = C_{Bvar2}^{E0}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$C_{Bvar4}^{E0} = C_{Bvar3}^{E0}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$C_{Bvar5}^{E0} = C_{Bvar4}^{E0}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$= C_{Bvar3}^{E0}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$= C_{Bvar3}^{E0}\left[I + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right] \qquad \text{(H-20)}$$

$$= C_{Bvar2}^{E0}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]\left[I + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$= C_{Bvar2}^{E0}\left[I + 3\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{9}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$C_{Bvar5}^{E0} - C_{Bvar3}^{E0} \approx 2\, C_{Bvar2}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

$$\underline{R}_{var5}^{E0} - \underline{R}_{var3}^{E0} = 2\,\underline{V}_{ref_0}^{E0}\, T_m + \left(C_{Bvar5}^{E0} - C_{Bvar3}^{E0}\right)\underline{l}^{Bref}$$

$$= 2\,\underline{V}_{ref_0}^{E0}\, T_m + 2\, C_{Bvar2}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]\underline{l}^{Bref}$$

$$\underline{V}_{var4}^{E0} = \frac{\underline{R}_{var5}^{E0} - \underline{R}_{var3}^{E0}}{2\, T_m} = \underline{V}_{ref_0}^{E0} + C_{Bvar2}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]\underline{l}^{Bref}$$

Summary of (H-14) to (H-20) Results:

$$\underline{V}^{E0}_{var_{-10}} = \underline{V}^{E0}_{ref_0}$$

$$\underline{V}^{E0}_{var_{-8}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-9}} \left[\left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) + \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right] \frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var_{-6}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-8}} \left[\left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) + 2 \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right] \frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var_{-4}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-6}} \left[\left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) + 2 \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right] \frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var_{-2}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-4}} \left[\left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) + 2 \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right] \frac{\underline{l}^{Bref}}{T_m} \qquad \text{(H-21)}$$

$$\underline{V}^{E0}_{var_0} = \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_{-2}} \left\langle \begin{array}{c} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] \\ + \frac{1}{2} \left\{ \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + 2\, \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right]^2 - \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right\} \end{array} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var_2} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) + 2 \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^2 \right] \frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var_4} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_2} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) + 2 \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^2 \right] \underline{l}^{Bref}$$

To obtain $\underline{V}^{E0}_{var_{2n-2}}$ in (H-1) as a function of $C^{E0}_{Bvar_{2n-2}}$ as is $\underline{V}^{E0}_{var_{2n}}$ in (H-21), find $C^{E0}_{Bvar_{2n}}$ as a function of $C^{E0}_{Bvar_{2n-2}}$. First note that

$$\left[I + \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) + \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right] \left[I - \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) + \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right]$$

$$= \left[\begin{array}{c} I - \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) + \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 + \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) - \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 + \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^3 \\ + \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 - \frac{1}{2} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^3 + \frac{1}{4} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^4 \end{array} \right] \qquad \text{(H-22)}$$

$$= I + \frac{1}{4} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^4$$

$$\left[I + \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) + \frac{1}{2} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^2 \right] \left[I - \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) + \frac{1}{2} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^2 \right] = I + \frac{1}{4} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^4$$

But

$$\left[I+\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]\left[I+\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]^{-1}=I$$

$$\left[I+\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\left[I+\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]^{-1}=I \tag{H-23}$$

Thus, to third order accuracy:

$$\left[I+\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]^{-1}\approx\left[I-\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]$$

$$\left[I+\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]^{-1}\approx\left[I-\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right] \tag{H-24}$$

Applying (H-24) to (32) obtains:

$$\text{For } m\le -9: \quad C_{Bvar_{m-1}}^{E_0}=C_{Bvar_{-9}}^{E_0}$$

$$\text{For } 0\ge m>-9:$$

$$C_{Bvar_{m-1}}^{E_0}\approx C_{Bvar_m}^{E_0}\left[I+\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]^{-1}$$

$$\approx C_{Bvar_m}^{E_0}\left[I-\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right] \tag{H-25}$$

$$\text{For } m>0: \quad C_{Bvar_{m-1}}^{E_0}\approx C_{Bvar_m}^{E_0}\left[I-\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)+\frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]$$

Successive application of (H-23) then finds $C_{Bvar_{m-2}}^{E_0}$ as a function of $C_{Bvar_m}^{E_0}$:

$$\text{For } m < -8: \quad C^{E0}_{Bvar_{m-1}} = C^{E0}_{Bvar_{-9}}$$

$$C^{E0}_{Bvar_{-9}} \approx C^{E0}_{Bvar_{-8}}\left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$\text{For } -2 \geq m > -8:$$

$$C^{E0}_{Bvar_{-8}} \approx C^{E0}_{Bvar_{-7}}\left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar_{-6}}\left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]\left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$\approx C^{E0}_{Bvar_{-6}}\left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar_{-6}}\left[I - 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$\vdots$$

$$C^{E0}_{Bvar_{m}} = C^{E0}_{Bvar_{m+2}}\left[I - 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right] \qquad \text{(H-26)}$$

$$\text{For } m = -1:$$

$$C^{E0}_{Bvar_{-1}} \approx C^{E0}_{Bvar_{0}}\left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar_{1}}\left[I - \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]\left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \right]$$

$$= C^{E0}_{Bvar_{1}}\begin{bmatrix} I - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 - \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) \\ + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \end{bmatrix}$$

$$= C^{E0}_{Bvar_{1}}\left\{ I - \left[\left(\Delta\underline{\alpha}^{Bvar}_{var} + \Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right] + \frac{1}{2}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var} + \Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]^2 \right\}$$

$$\text{For } m \geq 0:$$

$$C^{E0}_{Bvar_{m}} = C^{E0}_{Bvar_{m+2}}\left[I - 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \right]$$

With (H-26) and (H-21), $\underline{V}^{E0}_{var_{2n-2}}$ can be found from (H-1) as a function of $C^{E0}_{Bvar_{2n-2}}$.

For $n \le -4$: $\underline{V}^{E0}_{var_{2n-2}} = \underline{V}^{E0}_{ref_0}$

For $n = -3$:

$$\underline{V}^{E0}_{var_{2n-2}} = \underline{V}^{E0}_{var_{-8}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-9}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-8}}\left[I - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-8}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

For $n = -2$: (H-27)

$$\underline{V}^{E0}_{var_{2n-2}} = \underline{V}^{E0}_{var_{-6}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-8}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-6}}\left[I - 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-6}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

For $n = -1$:

$$\underline{V}^{E0}_{var_{2n-2}} = \underline{V}^{E0}_{var_{-4}} \approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-4}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

(Continued)

(H-27) Continued

For $n = 0$:

$$\underline{V}_{var_0}^{E0} = \underline{V}_{ref_0}^{E0} + \frac{1}{2} C_{Bvar-2}^{E0} \left\langle \begin{matrix} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] \\ + \frac{1}{2} \left\{ \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + 2\, \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right]^2 - \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right\} \end{matrix} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}_{ref_0}^{E0} + \frac{1}{2} C_{Bvar0}^{E0} \begin{bmatrix} I - 2\left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \\ + 2\left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \end{bmatrix} \left\langle \begin{matrix} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] \\ + \frac{1}{2} \left\{ \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + 2\, \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right]^2 - \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right\} \end{matrix} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}_{ref_0}^{E0} + \frac{1}{2} C_{Bvar0}^{E0} \left\langle \begin{matrix} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] \\ + \frac{1}{2} \left\{ \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + 2\, \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right]^2 - \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right\} \\ -2 \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] \\ + \text{higher than second order terms} \end{matrix} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}_{ref_0}^{E0} + \frac{1}{2} C_{Bvar0}^{E0} \left\langle \begin{matrix} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] \\ + \frac{1}{2} \left\{ \begin{matrix} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^2 + 4 \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) + 4 \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \\ - \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 - 4 \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] \end{matrix} \right\} \\ + \text{higher than second order terms} \end{matrix} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}_{ref_0}^{E0} + \frac{1}{2} C_{Bvar0}^{E0} \left\{ \begin{matrix} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] + \frac{1}{2} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^2 - \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right)^2 \right] \\ + \text{higher than second order terms} \end{matrix} \right\}$$

(Continued)

(H-27) Continued

For $n = 1$:

$$\underline{V}^{E0}_{var_{2n-2}} = \underline{V}^{E0}_{var_0} = \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar-2} \left\langle \begin{matrix} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ +\frac{1}{2}\left\{\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right\} \end{matrix} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar0} \begin{bmatrix} I - 2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) \\ +2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \end{bmatrix} \left\langle \begin{matrix} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ +\frac{1}{2}\left\{\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right\} \end{matrix} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar0} \left\langle \begin{matrix} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] + \frac{1}{2}\left\{\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right\} \\ -2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] + \text{higher than second order terms} \end{matrix} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar0} \left\langle \begin{matrix} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ +\frac{1}{2}\left\{ \begin{matrix} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 + 4\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + 4\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \\ -\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 - 4\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \end{matrix} \right\} \\ +\text{higher than second order terms} \end{matrix} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar0} \left\langle \begin{matrix} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] + \frac{1}{2}\left\{\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right\} \\ +\text{ higher than second order terms} \end{matrix} \right\rangle \frac{\underline{l}^{Bref}}{T_m}$$

For $n = 2$:

$$\underline{V}^{E0}_{var_{2n-2}} = \underline{V}^{E0}_{var_2} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar2}\left[I - 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 - 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

(Continued)

(H-27) Concluded

For $n = 3$:

$$\underline{V}^{E0}_{var\,2n-2} = \underline{V}^{E0}_{var\,4} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,4}\left[I - 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

For $n > 3$:

$$\underline{V}^{E0}_{var\,2n-2} \approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2n-2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

Summary of (H-27) Results:

$$\text{For } n \le -4: \quad \underline{V}^{E0}_{var\,2n-2} = \underline{V}^{E0}_{ref_0}$$

$$\underline{V}^{E0}_{var\,-8} \approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,-8}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var\,-6} \approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,-6}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var\,-4} \approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,-4}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var\,-2} \approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,-2}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m} \qquad \text{(H-28)}$$

$$\underline{V}^{E0}_{var\,0} = \underline{V}^{E0}_{ref_0} + \frac{1}{2}C^{E0}_{Bvar\,0}\left\{\begin{matrix}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] + \frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right] \\ + \text{higher than second order terms}\end{matrix}\right\}\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var\,2} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var\,4} \approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

For $n > 3$:

$$\underline{V}^{E0}_{var\,2n-2} \approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2n-2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

To find $\underline{V}^{E0}_{var\,2n} - \underline{V}^{E0}_{var\,2n-2}$ for (H-1), subtract (H-28) from (H-21):

$$\underline{V}_{var-10}^{E0} - \underline{V}_{var-12}^{E0} = \underline{V}_{ref_0}^{E0} - \underline{V}_{ref_0}^{E0} = 0$$

$$\underline{V}_{var-8}^{E0} - \underline{V}_{var-10}^{E0} = \underline{V}_{ref_0}^{E0} + C_{Bvar-9}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]\frac{\underline{\underline{l}}^{Bref}}{T_m} - \underline{V}_{ref_0}^{E0}$$

$$= C_{Bvar-9}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$\underline{V}_{var-6}^{E0} - \underline{V}_{var-8}^{E0} = \underline{V}_{ref_0}^{E0} + C_{Bvar-8}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$- \underline{V}_{ref_0}^{E0} - C_{Bvar-8}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \text{higher than second order terms}\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$= C_{Bvar-8}^{E0}\left[2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 - \text{higher than second order terms}\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$\underline{V}_{var-4}^{E0} - \underline{V}_{var-6}^{E0} = \underline{V}_{ref_0}^{E0} + C_{Bvar-6}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$- \underline{V}_{ref_0}^{E0} - C_{Bvar-6}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \text{higher than second order terms}\right]\frac{\underline{\underline{l}}^{Bref}}{T_m} \qquad \text{(H-29)}$$

$$= C_{Bvar-6}^{E0}\left[2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 - \text{higher than second order term}\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$\underline{V}_{var-2}^{E0} - \underline{V}_{var-4}^{E0} = \underline{V}_{ref_0}^{E0} + C_{Bvar-4}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + 2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$- \underline{V}_{ref_0}^{E0} - C_{Bvar-4}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \text{higher than second order terms}\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$= C_{Bvar-4}^{E0}\left[2\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2 - \text{higher than second order terms}\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$\underline{V}_{var0}^{E0} - \underline{V}_{var-2}^{E0} = \underline{V}_{ref_0}^{E0} + \frac{1}{2}C_{Bvar-2}^{E0}\left\langle \begin{matrix} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right] \\ +\frac{1}{2}\left\{\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]^2 - \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\right\} \end{matrix} \right\rangle \frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$- \underline{V}_{ref_0}^{E0} - C_{Bvar-2}^{E0}\left[\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right) + \text{higher than second order terms}\right]\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

$$= \frac{1}{2}C_{Bvar-2}^{E0}\left\langle\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right] - \text{higher than second order term}\right\rangle\frac{\underline{\underline{l}}^{Bref}}{T_m}$$

(Continued)

(H-29) Concluded

$$\underline{V}_{var_2}^{E0} - \underline{V}_{var_0}^{E0} = \underline{V}_{ref_0}^{E0} + C_{Bvar_0}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$-\underline{V}_{ref_0}^{E0} - \frac{1}{2}C_{Bvar_0}^{E0}\left\{\begin{matrix}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] + \frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right] \\ + \text{ higher than second order terms}\end{matrix}\right\}\frac{\underline{l}^{Bref}}{T_m}$$

$$= C_{Bvar_0}^{E0}\left\{\begin{matrix}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \\ -\frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] - \frac{1}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right] \\ - \text{ higher than second order terms}\end{matrix}\right\}\frac{\underline{l}^{Bref}}{T_m}$$

$$= C_{Bvar_0}^{E0}\left[\begin{matrix}\frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] + \frac{7}{4}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 + \frac{1}{4}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \\ - \text{ higher than second order term}\end{matrix}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}_{var_4}^{E0} - \underline{V}_{var_2}^{E0} = \underline{V}_{ref_0}^{E0} + C_{Bvar_2}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$-\underline{V}_{ref_0}^{E0} - C_{Bvar_2}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \text{ higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$= C_{Bvar_2}^{E0}\left[2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 - \text{ higher than second order term}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\approx C_{Bvar_2}^{E0}\, 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}_{var_6}^{E0} - \underline{V}_{var_4}^{E0} = \underline{V}_{ref_0}^{E0} + C_{Bvar_4}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + 2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$-\underline{V}_{ref_0}^{E0} - C_{Bvar_4}^{E0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \text{ higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\approx C_{Bvar_4}^{E0}\left[2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

Summary of (H-29) Results:

$$\underline{V}^{E0}_{var-10} - \underline{V}^{E0}_{var-12} = \underline{V}^{E0}_{ref_0} - \underline{V}^{E0}_{ref_0} = 0$$

$$\underline{V}^{E0}_{var-8} - \underline{V}^{E0}_{var-10} = C^{E0}_{Bvar-9}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var-6} - \underline{V}^{E0}_{var-8} = C^{E0}_{Bvar-8}\left[2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 - \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var-4} - \underline{V}^{E0}_{var-6} = C^{E0}_{Bvar-6}\left[2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 - \text{higher than second order term}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var-2} - \underline{V}^{E0}_{var-4} = C^{E0}_{Bvar-4}\left[2\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 - \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m} \qquad \text{(H-30)}$$

$$\underline{V}^{E0}_{var_0} - \underline{V}^{E0}_{var-2} = \frac{1}{2}C^{E0}_{Bvar-2}\left\langle\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] - \text{higher than second order term}\right\rangle\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var_2} - \underline{V}^{E0}_{var_0} = C^{E0}_{Bvar_0}\left[\begin{array}{c}\frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] + \frac{7}{4}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 + \frac{1}{4}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \\ - \text{higher than second order term}\end{array}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var_4} - \underline{V}^{E0}_{var_2} = C^{E0}_{Bvar_2}\left[2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 - \text{higher than second order term}\right]\frac{\underline{l}^{Bref}}{T_m}$$

$$\underline{V}^{E0}_{var_6} - \underline{V}^{E0}_{var_4} = C^{E0}_{Bvar_4}\left[2\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 - \text{higher than second order terms}\right]\frac{\underline{l}^{Bref}}{T_m}$$

Finding $B_{num_{2n-1}}$ And $B_{num_{2n}}$

To find $B_{num_{2n-1}}$ and $B_{num_{2n}}$, substitute A_{2n-1}, A_{2n} from (H-5), $\underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}}$ from (H-13), $\underline{V}^{E0}_{var_{2n-2}}$ from (H-28), and $\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}}$ from (H-30) into the (H-1) $B_{num_{2n-1}}$ and $B_{num_{2n}}$ equations:

For $n < -4$:

$$B_{num_{2n-1}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n-1}\left(\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$
$$+2\left(A_{2n-1} - I\right)\underline{g}^{E0}\,T_m^2 = -\,\underline{g}^{E0}_{avg}\,T_m^2 \qquad \text{(H-31)}$$

$$B_{num_{2n}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n}\left(\underline{V}^{E0}_{var_{2m}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$
$$+2\left(A_{2n} - I\right)\underline{g}^{E0}\,T_m^2 = \underline{g}^{E0}_{avg}\,T_m^2$$

For $n = -4$:

$$B_{num_{2n-1}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n-1}\left(\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$

$$+2\left(A_{2n-1} - I\right)\underline{g}^{E0}_{avg}\,T_m^2$$

$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + C^{E0}_{Bvar_{-9}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E0}_{ref_0}\,T_m$$

$$-\frac{1}{2}\left[I - \frac{1}{6}C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\right]C^{E0}_{Bvar_{-9}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref}$$

$$-\left[I + \frac{1}{6}C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\right]\underline{g}^{E0}_{avg}\,T_m^2$$

$$\approx C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} - \frac{1}{2}C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$

$$-\left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{-9}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

$$= \frac{1}{2}\,C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} - \left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{-9}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2 \qquad \text{(H-32)}$$

$$B_{num_{2n}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n}\left(\underline{V}^{E0}_{var_{2m}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$

$$+2\left(A_{2n} - I\right)\underline{g}^{E0}_{avg}\,T_m^2$$

$$= C^{E0}_{Bvar_{-9}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - \frac{3}{2}\,C^{E0}_{Bvar_{-9}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref}$$

$$+\,\underline{g}^{E0}_{avg}\,T_m^2$$

$$\approx C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} - \frac{3}{2}C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} + \underline{g}^{E0}_{avg}\,T_m^2$$

$$= -\frac{1}{2}C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} + \underline{g}^{E0}_{avg}\,T_m^2$$

For $n=-3$:

$$B_{num_{2n-1}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n-1}\left(\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$

$$+2\left(A_{2n-1}-I\right)\underline{g}^{E0}_{avg}\,T_m^2$$

$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + 2\,C^{E0}_{Bvar_{-8}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E0}_{ref_0}\,T_m$$

$$-\,2\,C^{E0}_{Bvar_{-8}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m}$$

$$-\left[I - \frac{1}{6}C^{E0}_{Bvar_{-7}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-7}}\right)^{-1}\right]C^{E0}_{Bvar_{-8}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\underline{l}^{Bref}$$

$$-\left[I + \frac{1}{6}C^{E0}_{Bvar_{-7}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-7}}\right)^{-1}\right]\underline{g}^{E0}_{avg}\,T_m^2$$

$$= C^{E0}_{Bvar_{-8}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\underline{l}^{Bref} - \left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{-8}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

(H-33)

$$B_{num_{2n}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n}\left(\underline{V}^{E0}_{var_{2m}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$

$$+2\left(A_{2n}-I\right)\underline{g}^{E0}\,T_m^2$$

$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + 2\,C^{E0}_{Bvar_{-8}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E0}_{ref_0}\,T_m$$

$$-2\,C^{E0}_{Bvar_{-8}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$

$$-3\left[I - \frac{1}{18}C^{E0}_{Bvar_{-8}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\right]C^{E0}_{Bvar_{-8}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\underline{l}^{Bref}$$

$$+\left[I - \frac{1}{6}C^{E0}_{Bvar_{-8}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-8}}\right)^{-1}\right]\underline{g}^{E0}_{avg}\,T_m^2$$

$$= -\,C^{E0}_{Bvar_{-8}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\underline{l}^{Bref} + \left\{I - \frac{1}{6}\left[\left(C^{E0}_{Bvar_{-8}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

For $0 > n > -3$:

$$B_{num_{2n-1}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n-1}\left(\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$

$$+2\left(A_{2n-1} - I\right)\underline{g}^{E0}_{avg}\,T_m^2$$

$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + 2\,C^{E0}_{Bvar_{2n-2}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E0}_{ref_0}\,T_m$$

$$-2\,C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m}$$

$$-\left[I - \frac{1}{6}C^{E0}_{Bvar_{2n-1}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\right]C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\underline{l}^{Bref}$$

$$-\left[I + \frac{1}{6}C^{E0}_{Bvar_{2n-1}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\right]\underline{g}^{E0}_{avg}\,T_m^2$$

$$\approx C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\underline{l}^{Bref} - \left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-2}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

(H-34)

$$B_{num_{2n}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n}\left(\underline{V}^{E0}_{var_{2m}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$

$$+2\left(A_{2n} - I\right)\underline{g}^{E0}_{avg}\,T_m^2$$

$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + 2\,C^{E0}_{Bvar_{2n-2}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E0}_{ref_0}\,T_m$$

$$-2\,C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$

$$-3\left[I - \frac{1}{18}C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\right]C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\underline{l}^{Bref}$$

$$+\left[I - \frac{1}{6}C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\right]\underline{g}^{E0}_{avg}\,T_m^2$$

$$= -\,C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\underline{l}^{Bref} + \left\{I - \frac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-2}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

For $n = 0$:

$$B_{num_{2n-1}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n-1}\left(\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$
$$+2\left(A_{2n-1} - I\right)\underline{g}^{E0}_{avg}\,T_m^2$$
$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + 2\,C^{E0}_{Bvar_{-2}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E0}_{ref_0}\,T_m$$
$$-2\,C^{E0}_{Bvar_{-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$
$$-\frac{1}{4}\left[I - \frac{1}{6}C^{E0}_{Bvar_{-1}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-1}}\right)^{-1}\right]C^{E0}_{Bvar_{-2}}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m}$$
$$-\left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{-1}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$
$$\approx -\frac{1}{4}C^{E0}_{Bvar_{-2}}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} - \left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{-2}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

(H-35)

$$B_{num_{2n}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n}\left(\underline{V}^{E0}_{var_{2m}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$
$$+2\left(A_{2n} - I\right)\underline{g}^{E0}_{avg}\,T_m^2$$
$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + 2\,C^{E0}_{Bvar_{-2}}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E0}_{ref_0}\,T_m$$
$$-2\,C^{E0}_{Bvar_{-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$
$$-\frac{3}{4}\left[I - \frac{1}{18}C^{E0}_{Bvar_{-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-2}}\right)^{-1}\right]\frac{1}{2}C^{E0}_{Bvar_{-2}}\left\langle \begin{array}{c} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ +\dfrac{1}{2}\left\{\begin{array}{c}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + 2\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2 \\ -\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\end{array}\right\}\end{array}\right\rangle\underline{l}^{Bref}$$
$$+\left[I - \frac{1}{6}C^{E0}_{Bvar_{-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-2}}\right)^{-1}\right]\underline{g}^{E0}_{avg}\,T_m^2$$
$$\approx -\frac{3}{4}\,C^{E0}_{Bvar_{-2}}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} + \left\{I - \frac{1}{6}\left[\left(C^{E0}_{Bvar_{-2}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

Note: Second order terms in (H-35) are neglected. If second order terms were included, $\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)$ products would appear which are too difficult to explain. For simplification, this article only carries the highest order $\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)$ or $\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)$ products.

For $n=1$:

$$B_{num_{2n-1}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n-1}\left(\underline{V}^{E0}_{var_{2n}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m$$

$$+2\left(A_{2n-1} - I\right)\underline{g}^{E0}_{avg}\,T_m^2$$

$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + 2\,C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E0}_{ref_0}\,T_m$$

$$-\,C^{E0}_{Bvar_0}\left\{\begin{array}{c}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ +\frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2\end{array}\right\}\underline{l}^{Bref}$$

$$-\frac{1}{4}\left[I - \frac{1}{6}C^{E0}_{Bvar_1}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_1}\right)^{-1}\right]C^{E0}_{Bvar_0}\left\{\begin{array}{c}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ +\frac{7}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \\ +\frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\end{array}\right\}\underline{l}^{Bref} \qquad \text{(H-36)}$$

$$-\left[I + \frac{1}{6}C^{E0}_{Bvar_1}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_1}\right)^{-1}\right]\underline{g}^{E0}_{avg}\,T_m^2$$

$$\approx 2\,C^{E0}_{Bvar_0}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\underline{l}^{Bref} - C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref}$$

$$-\frac{1}{4}\,C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} - \left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_1}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}\,T_m^2$$

$$= \frac{3}{4}C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} - \left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_0}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}\,T_m^2$$

(Continued)

(H-36) Concluded

$$B_{num_{2n}} \equiv \underline{R}^{E0}_{var_{2n}} - \underline{R}^{E0}_{var_{2n-2}} - 2\,\underline{V}^{E0}_{var_{2n-2}}\,T_m - A_{2n}\left(\underline{V}^{E0}_{var_{2m}} - \underline{V}^{E0}_{var_{2n-2}}\right)T_m + 2\left(A_{2n} - I\right)\underline{g}^{E0}_{avg}\,T_m^2$$

$$= 2\,\underline{V}^{E0}_{ref_0}\,T_m + 2\,C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E0}_{ref_0}\,T_m$$

$$- C^{E0}_{Bvar_0}\left\{\begin{array}{c}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] - \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ + \frac{1}{2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]^2\end{array}\right\}\underline{l}^{Bref}$$

$$-\frac{3}{2}\left[I - \frac{1}{18}C^{E0}_{Bvar_0}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_0}\right)^{-1}\right]\frac{1}{2}C^{E0}_{Bvar_0}\left\{\begin{array}{c}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \\ + \frac{7}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \\ + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2\end{array}\right\}\underline{l}^{Bref}$$

$$+\left\{I - \frac{1}{6}\left[\left(C^{E0}_{Bvar_0}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

$$\approx 2\,C^{E0}_{Bvar_0}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\underline{l}^{Bref} - C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref}$$

$$-\frac{3}{4}C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} + \left\{I - \frac{1}{6}\left[\left(C^{E0}_{Bvar_0}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

$$= \frac{1}{4}C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} + \left\{I - \frac{1}{6}\left[\left(C^{E0}_{Bvar_0}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2$$

For $n > 1$:

$$B_{num_{2n-1}} \equiv \underline{R}^{E_0}_{var_{2n}} - \underline{R}^{E_0}_{var_{2n-2}} - 2\,\underline{V}^{E_0}_{var_{2n-2}}\,T_m - A_{2n-1}\left(\underline{V}^{E_0}_{var_{2n}} - \underline{V}^{E_0}_{var_{2n-2}}\right)T_m$$

$$+2\left(A_{2n-1} - I\right)\underline{g}^{E_0}_{avg}\,T_m^2$$

$$= 2\,\underline{V}^{E_0}_{ref_0}\,T_m + 2\,C^{E_0}_{Bvar_{2n-2}}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E_0}_{ref_0}\,T_m$$

$$-2\,C^{E_0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$

$$-\frac{1}{2}\left[I - \frac{1}{6}C^{E_0}_{Bvar_{2n-1}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E_0}_{Bvar_{2n-1}}\right)^{-1}\right]2\,C^{E_0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\underline{l}^{Bref}$$

$$-\left\{I + \frac{1}{6}\left[\left(C^{E_0}_{Bvar_{2n-1}}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E_0}_{avg}\,T_m^2$$

$$\approx C^{E_0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\underline{l}^{Bref} - \left\{I + \frac{1}{6}\left[\left(C^{E_0}_{Bvar_{2n-1}}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E_0}_{avg}\,T_m^2$$

$$\approx C^{E_0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\underline{l}^{Bref} - \left\{I + \frac{1}{6}\left[\left(C^{E_0}_{Bvar_{2n-2}}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E_0}_{avg}\,T_m^2$$

(H-37)

$$B_{num_{2n}} \equiv \underline{R}^{E_0}_{var_{2n}} - \underline{R}^{E_0}_{var_{2n-2}} - 2\,\underline{V}^{E_0}_{var_{2n-2}}\,T_m - A_{2n}\left(\underline{V}^{E_0}_{var_{2m}} - \underline{V}^{E_0}_{var_{2n-2}}\right)T_m$$

$$+2\left(A_{2n} - I\right)\underline{g}^{E_0}_{avg}\,T_m^2$$

$$= 2\,\underline{V}^{E_0}_{ref_0}\,T_m + 2\,C^{E_0}_{Bvar_{2n-2}}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\right]\underline{l}^{Bref} - 2\,\underline{V}^{E_0}_{ref_0}\,T_m$$

$$-2\,C^{E_0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$

$$-\frac{3}{2}\left[I - \frac{1}{18}C^{E_0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E_0}_{Bvar_{2n-2}}\right)^{-1}\right]2\,C^{E_0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\underline{l}^{Bref}$$

$$+\left\{I - \frac{1}{6}\left[\left(C^{E_0}_{Bvar_{2n-2}}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E_0}_{avg}\,T_m^2$$

$$\approx -\,C^{E_0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\underline{l}^{Bref} + \left\{I - \frac{1}{6}\left[\left(C^{E_0}_{Bvar_{2n-2}}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E_0}_{avg}\,T_m^2$$

Summary of (H-31) – (H-37) Results:

For $n < -4$:

$$B_{num_{2n-1}} = -\underline{g}^{E0}\, T_m^2 \qquad B_{num_{2n}} = \underline{g}^{E0}\, T_m^2$$

For $n = -4$:

$$B_{num_{2n-1}} = \frac{1}{2} C_{Bvar_{-9}}^{E0} \left(\Delta\underline{\alpha}_{var}^{Bvar} \times \right) \underline{l}^{Bref} - \left\{ I + \frac{1}{6} \left[\left(C_{Bvar_{-9}}^{E0}\, \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

$$B_{num_{2n}} = -\frac{1}{2} C_{Bvar_{-9}}^{E0} \left(\Delta\underline{\alpha}_{var}^{Bvar} \times \right) \underline{l}^{Bref} + \underline{g}_{avg}^{E0}\, T_m^2$$

For $n = -3$:

$$B_{num_{2n-1}} = C_{Bvar_{-8}}^{E0} \left(\Delta\underline{\alpha}_{var}^{Bvar} \times \right)^2 \underline{l}^{Bref} - \left\{ I + \frac{1}{6} \left[\left(C_{Bvar_{-8}}^{E0}\, \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

$$B_{num_{2n}} = -C_{Bvar_{-8}}^{E0} \left(\Delta\underline{\alpha}_{var}^{Bvar} \times \right)^2 \underline{l}^{Bref} + \left\{ I - \frac{1}{6} \left[\left(C_{Bvar_{-8}}^{E0}\, \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

For $0 > n > -3$: (H-38)

$$B_{num_{2n-1}} = C_{Bvar_{2n-2}}^{E0} \left(\Delta\underline{\alpha}_{var}^{Bvar} \times \right)^2 \underline{l}^{Bref} - \left\{ I + \frac{1}{6} \left[\left(C_{Bvar_{2n-2}}^{E0}\, \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

$$B_{num_{2n}} = -C_{Bvar_{2n-2}}^{E0} \left(\Delta\underline{\alpha}_{var}^{Bvar} \times \right)^2 \underline{l}^{Bref} + \left\{ I - \frac{1}{6} \left[\left(C_{Bvar_{2n-2}}^{E0}\, \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

For $n = 0$:

$$B_{num_{2n-1}} \approx -\frac{1}{4} C_{Bvar_{-2}}^{E0} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \underline{l}^{Bref} - \left\{ I + \frac{1}{6} \left[\left(C_{Bvar_{-2}}^{E0}\, \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

$$B_{num_{2n}} \approx -\frac{3}{4} C_{Bvar_{-2}}^{E0} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \underline{l}^{Bref} + \left\{ I - \frac{1}{6} \left[\left(C_{Bvar_{-2}}^{E0}\, \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

For $n = 1$:

$$B_{num_{2n-1}} = \frac{3}{4} C_{Bvar_{0}}^{E0} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \underline{l}^{Bref} - \left\{ I + \frac{1}{6} \left[\left(C_{Bvar_{0}}^{E0}\, \Delta\underline{\alpha}'^{Bvar}_{var} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

$$B_{num_{2n}} = \frac{1}{4} C_{Bvar_{0}}^{E0} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar} \right) \times \right] \underline{l}^{Bref} + \left\{ I - \frac{1}{6} \left[\left(C_{Bvar_{0}}^{E0}\, \Delta\underline{\alpha}'^{Bvar}_{var} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

For $n > 1$:

$$B_{num_{2n-1}} \approx C_{Bvar_{2n-2}}^{E0} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right)^2 \underline{l}^{Bref} - \left\{ I + \frac{1}{6} \left[\left(C_{Bvar_{2n-2}}^{E0}\, \Delta\underline{\alpha}'^{Bvar}_{var} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

$$B_{num_{2n}} \approx -C_{Bvar_{2n-2}}^{E0} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right)^2 \underline{l}^{Bref} + \left\{ I - \frac{1}{6} \left[\left(C_{Bvar_{2n-2}}^{E0}\, \Delta\underline{\alpha}'^{Bvar}_{var} \right) \times \right] \right\} \underline{g}_{avg}^{E0}\, T_m^2$$

Finding Specific Force $\Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar}$ And $\Delta \underline{\upsilon}_{var_{2n}}^{Bvar}$

Specific force $\Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar}$ and $\Delta \underline{\upsilon}_{var_{2n}}^{Bvar}$ are obtained by substituting $B_{den_{2n-1}}^{-1}$, $B_{den_{2n}}^{-1}$ from (H-10) and $B_{num_{2n-1}}$, $B_{num_{2n}}$ from (H-38) into the (H-1) $\Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar}$, $\Delta \underline{\upsilon}_{var_{2n}}^{Bvar}$ formulas:

For $n < -4$:

$$\Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar} = B_{den_{2n-1}}^{-1} \, B_{num_{2n-1}} = -\left(C_{Bvar_{-9}}^{E_0} \right)^{-1} \underline{g}_{avg}^{E_0} \, T_m \qquad \text{(H-39)}$$

$$\Delta \underline{\upsilon}_{var_{2n}}^{Bvar} = B_{den_{2n}}^{-1} \, B_{num_{2n}} = -\left(C_{Bvar_{-9}}^{E_0} \right)^{-1} \underline{g}_{avg}^{E_0} \, T_m$$

For $n = -4$:

$$\Delta\underline{\upsilon}^{Bvar}_{var_{2n-1}} = B^{-1}_{den_{2n-1}}\, B_{num_{2n-1}}$$

$$= \left[I - \frac{1}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\left\langle \begin{array}{c} \frac{1}{2}\, C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} \\ -\left\{ I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{-9}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\, T_m \end{array}\right\rangle$$

$$\approx \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{-9}}\right)^{-1}\left\{ I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{-9}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\, T_m$$

$$+ \frac{1}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}\, T_m - \frac{1}{6}\left(C^{E0}_{Bvar_{-9}}\right)^{-1} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$+ \frac{1}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m - \frac{1}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left[I + \frac{1}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{4} - \frac{1}{3}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m \tag{H-40}$$

$$\Delta\underline{\upsilon}^{Bvar}_{var_{2n}} = B^{-1}_{den_{2n}}\, B_{num_{2n}}$$

$$= -\left[I - \frac{7}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\left[-\frac{1}{2} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} + \underline{g}^{E0}_{avg}\right]$$

$$\approx \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg} + \frac{7}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left[I - \frac{7}{12}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{4} + \frac{1}{3}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

For $0 > n > -4$:

$$\Delta\underline{\upsilon}^{Bvar}_{var_{2n-1}} = B^{-1}_{den_{2n-1}}\, B_{num_{2n-1}}$$

$$= \left[I - \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\left\langle \begin{array}{c} C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^{2} \dfrac{\underline{l}^{Bref}}{T_m} \\ -\left\{I + \dfrac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-2}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\, T_m \end{array}\right\rangle$$

$$\approx \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^{2}\frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-2}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\, T_m$$

$$+\frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^{2}\frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}\, T_m + \left(\frac{1}{2} - \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^{2}\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2} - \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

(H-41)

$$\Delta\underline{\upsilon}^{Bvar}_{var_{2n}} = B^{-1}_{den_{2n}}\, B_{num_{2n}}$$

$$= -\left[I - \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\left\langle \begin{array}{c} -\,C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^{2} \dfrac{\underline{l}^{Bref}}{T_m} \\ +\left\{I - \dfrac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-2}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}\, T_m \end{array}\right\rangle$$

$$\approx \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^{2}\frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\left\{I - \frac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-2}}\,\Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\, T_m$$

$$+\frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$\approx \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^{2}\frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m + \left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^{2}\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

For $n = 0$:

$$\Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar} = B_{den_{2n-1}}^{-1} B_{num_{2n-1}}$$

$$= \left[I - \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \left\langle \begin{matrix} -\frac{1}{4} C_{Bvar_{-2}}^{E0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar} \right) \times \right] \underline{l}^{Bref} \\ -\left\{ I + \frac{1}{6} \left[\left(C_{Bvar_{-2}}^{E0} \Delta \underline{\alpha}_{var}^{Bvar} \right) \times \right] \right\} \underline{g}_{avg}^{E0} T_m^2 \end{matrix} \right\rangle / T_m$$

$$\approx -\frac{1}{4} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m}$$

$$- \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \left\{ I + \frac{1}{6} \left[\left(C_{Bvar_{-2}}^{E0} \Delta \underline{\alpha}_{var}^{Bvar} \right) \times \right] \right\} \underline{g}_{avg}^{E0} T_m + \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m$$

$$\approx -\frac{1}{4} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} - \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m$$

(H-42)

$$-\frac{1}{6} \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \left[\left(C_{Bvar_{-2}}^{E0} \Delta \underline{\alpha}_{var}^{Bvar} \right) \times \right] \underline{g}^{E0} T_m + \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m$$

$$\approx -\frac{1}{4} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} - \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m$$

$$-\frac{1}{6} \left(C_{Bvar_{-2}}^{E0} \right)^{-1} C_{Bvar_{-2}}^{E0} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m + \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m$$

$$\approx -\frac{1}{4} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} - \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}^{E0} T_m$$

$$-\frac{1}{6} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m + \frac{1}{2} \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m$$

$$= -\frac{1}{4} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2} - \frac{1}{6} \right) \left(\Delta \underline{\alpha}_{var}^{Bvar} \times \right) \right] \left(C_{Bvar_{-2}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m$$

(Continued)

(H-42) Concluded

$$\Delta \underline{\upsilon}_{var_{2n}}^{Bvar} = B_{den_{2n}}^{-1} \, B_{num_{2n}}$$

$$= -\left[I - \frac{1}{2}\left(\Delta \underline{\alpha}_{var}^{Bvar} \times\right)\right]\left(C_{Bvar_{-1}}^{E0}\right)^{-1} \left\langle \begin{array}{l} -\frac{3}{4} \, C_{Bvar_{-2}}^{E0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar}\right)\times\right] \underline{l}^{Bref} \\ + \left\{ I - \frac{1}{6}\left[\left(C_{Bvar_{-2}}^{E0} \, \Delta \underline{\alpha}_{var}^{Bvar}\right)\times\right]\right\} \underline{g}^{E0} \, T_m^2 \end{array} \right\rangle / \, T_m$$

$$= \frac{3}{4}\left(C_{Bvar_{-1}}^{E0}\right)^{-1} C_{Bvar_{-2}}^{E0} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar}\right)\times\right] \frac{\underline{l}^{Bref}}{T_m}$$

$$-\left(C_{Bvar_{-1}}^{E0}\right)^{-1} \left\{ I - \frac{1}{6}\left[\left(C_{Bvar_{-2}}^{E0} \, \Delta \underline{\alpha}_{var}^{Bvar}\right)\times\right]\right\} \underline{g}_{avg}^{E0} \, T_m + \frac{1}{2}\left(\Delta \underline{\alpha}_{var}^{Bvar} \times\right)\left(C_{Bvar_{-1}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0} \, T_m$$

$$\approx \frac{3}{4}\left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar}\right)\times\right] \frac{\underline{l}^{Bref}}{T_m} - \left(C_{Bvar_{-1}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0} \, T_m$$

$$+ \frac{1}{6}\left(\Delta \underline{\alpha}_{var}^{Bvar} \times\right)\left(C_{Bvar_{-1}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0} \, T_m + \frac{1}{2}\left(\Delta \underline{\alpha}_{var}^{Bvar} \times\right)\left(C_{Bvar_{-1}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0} \, T_m$$

$$= \frac{3}{4}\left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}_{var}^{Bvar}\right)\times\right] \frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta \underline{\alpha}_{var}^{Bvar} \times\right)\right]\left(C_{Bvar_{-1}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0} \, T_m$$

For $n=1$:

$$\Delta\underline{\upsilon}^{Bvar}_{var\,2n-1} = B^{-1}_{den\,2n-1}\,B_{num\,2n-1}$$

$$= \left[I - \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar\,0}\right)^{-1}\left\langle \begin{array}{l} \frac{3}{4}C^{E0}_{Bvar\,0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} \\ -\left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar\,0}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2 \end{array}\right\rangle / T_m$$

$$\approx \frac{3}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar\,0}\right)^{-1}\underline{g}^{E0}_{avg}\,T_m$$
$$- \left(C^{E0}_{Bvar\,0}\right)^{-1}\left\{I + \frac{1}{6}\left[\left(C^{E0}_{Bvar\,0}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m$$

$$\approx \frac{3}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar\,0}\right)^{-1}\underline{g}^{E0}_{avg}\,T_m$$
$$- \left(C^{E0}_{Bvar\,0}\right)^{-1}\underline{g}^{E0}_{avg}\,T_m - \frac{1}{6}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar\,0}\right)^{-1}\underline{g}^{E0}_{avg}\,T_m$$

$$= \frac{3}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2} - \frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar\,0}\right)^{-1}\underline{g}^{E0}_{avg}\,T_m$$

(H-43)

$$\Delta\underline{\upsilon}^{Bvar}_{var\,2n} = B^{-1}_{den\,2n}\,B_{num\,2n}$$

$$= -\left[I - \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar\,1}\right)^{-1}\left\langle \begin{array}{l} \frac{1}{4}C^{E0}_{Bvar\,0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} \\ +\left\{I - \frac{1}{6}\left[\left(C^{E0}_{Bvar\,0}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m^2 \end{array}\right\rangle / T_m$$

$$\approx -\frac{1}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar\,1}\right)^{-1}\underline{g}^{E0}_{avg}\,T_m$$
$$- \left(C^{E0}_{Bvar\,1}\right)^{-1}\left\{I - \frac{1}{6}\left[\left(C^{E0}_{Bvar\,0}\,\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\,T_m$$

$$\approx -\frac{1}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar\,1}\right)^{-1}\underline{g}^{E0}_{avg}\,T_m$$

For $n > 1$:

$$\Delta\underline{\upsilon}_{var_{2n-1}}^{Bvar} = B_{den_{2n-1}}^{-1}\, B_{num_{2n-1}}$$

$$= \left[I - \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} \left\langle \begin{array}{c} C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \underline{l}^{Bref} \\ -\left\{ I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-1}}\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\, T_m^2 \end{array} \right\rangle / T_m$$

$$\approx \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\left\{ I + \frac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-1}}\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\, T_m$$

$$+\frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$\approx \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2} - \frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg}\, / T_m$$

(H-44)

$$\Delta\underline{\upsilon}_{var_{2n}}^{Bvar} = B_{den_{2n}}^{-1}\, B_{num_{2n}}$$

$$= -\left[I - \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1} \left\langle \begin{array}{c} -\,C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \underline{l}^{Bref} \\ +\left\{ I - \frac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-2}}\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\, T_m^2 \end{array} \right\rangle / T_m$$

$$\approx \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \frac{\underline{l}^{Bref}}{T_m} - \left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\left\{ I - \frac{1}{6}\left[\left(C^{E0}_{Bvar_{2n-2}}\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\right\}\underline{g}^{E0}_{avg}\, T_m$$

$$+\frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$\approx \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

Summary of (H-39) – (H-44) Specific Force Results:

For $n < -4$:

$$\Delta\underline{\upsilon}_{var_{2n-1}}^{Bvar} = -\left(C_{Bvar_{-9}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0}\, T_m \qquad \Delta\underline{\upsilon}_{var_{2n}}^{Bvar} = -\left(C_{Bvar_{-9}}^{E0}\right)^{-1} \underline{g}_{avg}^{E0}\, T_m$$

For $n = -4$:

$$\Delta\underline{\upsilon}_{var_{2n-1}}^{Bvar} = \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{4}-\frac{1}{3}\right)\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)\right]\left(C_{Bvar_{-9}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

$$\Delta\underline{\upsilon}_{var_{2n}}^{Bvar} = \frac{1}{2}\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{4}+\frac{1}{3}\right)\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)\right]\left(C_{Bvar_{-9}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

For $0 > n > -4$:

$$\Delta\underline{\upsilon}_{var_{2n-1}}^{Bvar} = \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2}-\frac{1}{6}\right)\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)\right]\left(C_{Bvar_{2n-2}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

$$\Delta\underline{\upsilon}_{var_{2n}}^{Bvar} = \left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)^2\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2}+\frac{1}{6}\right)\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)\right]\left(C_{Bvar_{2n-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

For $n = 0$:

$$\Delta\underline{\upsilon}_{var_{2n-1}}^{Bvar} = -\frac{1}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2}-\frac{1}{6}\right)\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)\right]\left(C_{Bvar_{-2}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

$$\Delta\underline{\upsilon}_{var_{2n}}^{Bvar} = \frac{3}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2}+\frac{1}{6}\right)\left(\Delta\underline{\alpha}_{var}^{Bvar}\times\right)\right]\left(C_{Bvar_{-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m \qquad \text{(H-45)}$$

For $n = 1$:

$$\Delta\underline{\upsilon}_{var_{2n-1}}^{Bvar} = \frac{3}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2}-\frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C_{Bvar_0}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

$$\Delta\underline{\upsilon}_{var_{2n}}^{Bvar} = -\frac{1}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}_{var}^{Bvar}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2}+\frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C_{Bvar_1}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

For $n > 1$:

$$\Delta\underline{\upsilon}_{var_{2n-1}}^{Bvar} = \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2}-\frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C_{Bvar_{2n-2}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

$$\Delta\underline{\upsilon}_{var_{2n}}^{Bvar} = \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{2}+\frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C_{Bvar_{2n-1}}^{E0}\right)^{-1}\underline{g}_{avg}^{E0}\, T_m$$

Recognizing that the inverse of a direction cosine matrix equals its transpose, the $\left(C_{Bvar_n}^{E0}\right)^{-1}$ terms in (H-45) become $C_{E0_n}^{Bvar}$, generating (49) in [1].

VELOCITY DETERMINATION

Approximating gravity for test example conditions as the constant $\underline{g}_{avg}^{E0}$, velocity from (3) with $m = 2n$ becomes at time instants *m*-1 and *m*:

$$\begin{aligned} \underline{V}_{var_{2n-1}}^{E0} &= \underline{V}_{var_{2n-2}}^{E0} + C_{Bvar_{2n-2}}^{E0} \, G_{Vvar_{2n-1}} \Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar} + \underline{g}_{avg}^{E0} \, T_m \\ \underline{V}_{var_{2n}}^{E0} &= \underline{V}_{var_{2n-1}}^{E0} + C_{Bvar_{2n-1}}^{E0} \, G_{Vvar_{2n}} \Delta \underline{\upsilon}_{var_{2n}}^{Bvar} + \underline{g}_{avg}^{E0} \, T_m \end{aligned} \qquad \text{(H-46)}$$

With initial velocity at $\underline{V}_{ref_0}^{E0}$, (H-45) for $\Delta \underline{\upsilon}_{var}^{Bvar}$ specific force, (32) for C_{Bvar}^{E0} attitude, and (33) for G_{Vvar}, (H-46) becomes to first order accuracy for velocity:

For $n < -4$:

$$\begin{aligned} \underline{V}_{var_{2n-1}}^{E0} &= \underline{V}_{var_{2n-2}}^{E0} + C_{Bvar_{2n-2}}^{E0} \, G_{Vvar_{2n-1}} \Delta \underline{\upsilon}_{var_{2n-1}}^{Bvar} + \underline{g}_{avg}^{E0} \, T_m \\ &= \underline{V}_{ref_0}^{E0} - C_{Bvar_{-9}}^{E0} \, I \left(C_{Bvar_{-9}}^{E0} \right)^{-1} \underline{g}^{E0} \, T_m + \underline{g}^{E0} \, T_m = \underline{V}_{ref_0}^{E0} \\ \underline{V}_{var_{2n}}^{E0} &= \underline{V}_{var_{2n-1}}^{E0} + C_{Bvar_{2n-1}}^{E0} \, G_{Vvar_{2n}} \Delta \underline{\upsilon}_{var_{2n}}^{Bvar} + \underline{g}_{avg}^{E0} \, T_m \\ &= \underline{V}_{ref_0}^{E0} - C_{Bvar_{2n-1}}^{E0} \, I \left(C_{Bvar_{-9}}^{E0} \right)^{-1} \underline{g}^{E0} \, T_m + \underline{g}_{avg}^{E0} \, T_m = \underline{V}_{ref_0}^{E0} \end{aligned} \qquad \text{(H-47)}$$

For $n = -4$:

$$\underline{V}^{E0}_{var_{2n-1}} = \underline{V}^{E0}_{var_{2n-2}} + C^{E0}_{Bvar_{2n-2}}\, G_{Vvar_{2n-1}}\, \Delta\underline{\upsilon}^{Bvar}_{var_{2n-1}} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{var_{-9}} = \underline{V}^{E0}_{var_{-10}} + C^{E0}_{Bvar_{-10}}\, G_{Vvar_{-9}}\, \Delta\underline{\upsilon}^{Bvar}_{var_{-9}} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-9}}\, I \left\{ \begin{array}{c} \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\frac{1}{4}-\frac{1}{3}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}\, T_m \end{array} \right\} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} + \left(\frac{1}{4}-\frac{1}{3}\right) C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{12} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m \tag{H-48}$$

$$\underline{V}^{E0}_{var_{2n}} = \underline{V}^{E0}_{var_{-8}} = \underline{V}^{E0}_{var_{-9}} + C^{E0}_{Bvar_{-9}}\, G_{Vvar_{-8}}\, \Delta\underline{\upsilon}^{Bvar}_{var_{2n}} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{12} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$+ C^{E0}_{Bvar_{-9}}\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left\{ \begin{array}{c} \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\frac{1}{4}+\frac{1}{3}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}\, T_m \end{array} \right\} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{12} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$+ C^{E0}_{Bvar_{-9}}\left\{\frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \left[I - \left(\frac{1}{4}+\frac{1}{3}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m\right\}$$

$$-\frac{1}{2} C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}\, T_m + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m}$$

For $0 > n > -4$:

$$\underline{V}^{E0}_{var\,2n-1} = \underline{V}^{E0}_{var\,2n-2} + C^{E0}_{Bvar\,2n-2}\, G_{Vvar\,2n-1}\, \Delta\underline{\upsilon}^{Bvar}_{var\,2n-1} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2n-3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m}$$

$$- C^{E0}_{Bvar\,2n-2}\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left[I - \left(\frac{1}{2} - \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar\,2n-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m + \underline{g}^{E0}_{avg}\, T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2n-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{2}C^{E0}_{Bvar\,2n-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar\,2n-2}\right)^{-1}\underline{g}^{E0}\, T_m$$

$$- C^{E0}_{Bvar\,2n-2}\left[-\left(\frac{1}{2} - \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar\,2n-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2n-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6}C^{E0}_{Bvar\,2n-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar\,2n-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

(H-49)

$$\underline{V}^{E0}_{var\,2n} = \underline{V}^{E0}_{var\,2n-1} + C^{E0}_{Bvar\,2n-1}\, G_{Vvar\,2n}\, \Delta\underline{\upsilon}^{Bvar}_{var\,2n} + \underline{g}^{E0}_{avg}\, T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2n-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6}C^{E0}_{Bvar\,2n-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar\,2n-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$- C^{E0}_{Bvar\,2n-1}\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left[I - \left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar\,2n-1}\right)^{-1}\underline{g}^{E0}\, T_m + \underline{g}^{E0}_{avg}\, T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2n-1}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6}C^{E0}_{Bvar\,2n-1}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar\,2n-1}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$- \frac{1}{2}C^{E0}_{Bvar\,2n-1}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar\,2n-1}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$- C^{E0}_{Bvar\,2n-1}\left[-\left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar\,2n-1}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar\,2n-1}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m}$$

For $n = 0$:

$$\underline{V}^{E0}_{var\,2n-1} = \underline{V}^{E0}_{var-1} = \underline{V}^{E0}_{var\,2n-2} + C^{E0}_{Bvar\,2n-2}\, G_{Vvar\,2n-1}\, \Delta\underline{\upsilon}^{Bvar}_{var\,2n-1} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{var-2} + C^{E0}_{Bvar-2}\, G_{Vvar-1}\, \Delta\underline{\upsilon}^{Bvar}_{var-1} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar-3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m}$$

$$+ C^{E0}_{Bvar\,2n-2}\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left\{\begin{array}{c} -\dfrac{1}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} \\ -\left[I - \left(\dfrac{1}{2} - \dfrac{1}{6}\right)\right]\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m \end{array}\right\} + \underline{g}^{E0}_{avg}\, T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{2}C^{E0}_{Bvar-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2$$

$$- C^{E0}_{Bvar-2}\left\{\frac{1}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} - \left(\frac{1}{2} - \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m\right\}$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{4}C^{E0}_{Bvar-2}\left[\left(5\,\Delta\underline{\alpha}^{Bvar}_{var} - \Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6}C^{E0}_{Bvar-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m$$

(H-50)

$$\underline{V}^{E0}_{var\,2n} = \underline{V}^{E0}_{var\,0} = \underline{V}^{E0}_{var\,2n-1} + C^{E0}_{Bvar\,2n-1}\, G_{Vvar\,2n}\, \Delta\underline{\upsilon}^{Bvar}_{var\,2n} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{var-1} + C^{E0}_{Bvar-1}\, G_{Vvar\,0}\, \Delta\underline{\upsilon}^{Bvar}_{var\,0} + \underline{g}^{E0}_{avg}\, T_m$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{4}C^{E0}_{Bvar-2}\left[\left(5\,\Delta\underline{\alpha}^{Bvar}_{var} - \Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6}C^{E0}_{Bvar-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-2}\right)^{-1}\underline{g}^{E0}\, T_m^2$$

$$+ C^{E0}_{Bvar-1}\left[I + \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left\{\begin{array}{c} \dfrac{3}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} \\ -\left[I - \left(\dfrac{1}{2} + \dfrac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar-1}\right)^{-1}\underline{g}^{E0}\, T_m^2 \end{array}\right\} + \underline{g}^{E0}_{avg}\, T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + \frac{1}{4}C^{E0}_{Bvar-1}\left[\left(5\,\Delta\underline{\alpha}^{Bvar}_{var} - \Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6}C^{E0}_{Bvar-1}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-1}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2$$

$$- \frac{1}{2}C^{E0}_{Bvar-1}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-1}\right)^{-1}\underline{g}^{E0}\, T_m^2 + \frac{3}{4}C^{E0}_{Bvar-1}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref}$$

$$+ C^{E0}_{Bvar-1}\left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-1}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2}C^{E0}_{Bvar-1}\left[\left(\Delta\underline{\alpha}^{Bvar}_{var} + \Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m}$$

For $n = 1$:

$$\underline{V}^{E0}_{var_{2n-1}} = \underline{V}^{E0}_{var_1} = \underline{V}^{E0}_{var_{2n-2}} + C^{E0}_{Bvar_{2n-2}} \, G_{Vvar_{2n-1}} \, \Delta\underline{\upsilon}^{Bvar}_{var_{2n-1}} + \underline{g}^{E0}_{avg} \, T_m$$

$$= \underline{V}^{E0}_{var_0} + C^{E0}_{Bvar_0} \, G_{Vvar_1} \, \Delta\underline{\upsilon}^{Bvar}_{var_1} + \underline{g}^{E0}_{avg} \, T_m$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_{-1}} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m}$$

$$+ C^{E0}_{Bvar_0} \left[I + \frac{1}{2} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \right] \left\{ \begin{array}{c} \frac{3}{4} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} \\ - \left[I - \left(\frac{1}{2} - \frac{1}{6} \right) \right] \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_0} \right)^{-1} \underline{g}^{E0} \, T_m \end{array} \right\} + \underline{g}^{E0}_{avg} \, T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_0} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{2} C^{E0}_{Bvar_0} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_0} \right)^{-1} \underline{g}^{E0}_{avg} \, T_m$$

$$+ \frac{3}{4} C^{E0}_{Bvar_0} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} + C^{E0}_{Bvar_0} \left(\frac{1}{2} - \frac{1}{6} \right) \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_0} \right)^{-1} \underline{g}^{E0}_{avg} \, T_m$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{4} C^{E0}_{Bvar_0} \left[\left(5 \, \Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_0} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_0} \right)^{-1} \underline{g}^{E0}_{avg} \, T_m$$

(H-51)

$$\underline{V}^{E0}_{var_{2n}} = \underline{V}^{E0}_{var_2} = \underline{V}^{E0}_{var_{2n-1}} + C^{E0}_{Bvar_{2n-1}} \, G_{Vvar_{2n}} \, \Delta\underline{\upsilon}^{Bvar}_{var_{2n}} + \underline{g}^{E0}_{avg} \, T_m$$

$$= \underline{V}^{E0}_{var_1} + C^{E0}_{Bvar_1} \, G_{Vvar_2} \, \Delta\underline{\upsilon}^{Bvar}_{var_2} + \underline{g}^{E0}_{avg} \, T_m$$

$$= \underline{V}^{E0}_{ref_0} + \frac{1}{4} C^{E0}_{Bvar_0} \left[\left(5 \, \Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_0} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_0} \right)^{-1} \underline{g}^{E0}_{avg} \, T_m$$

$$+ C^{E0}_{Bvar_1} \left[I + \frac{1}{2} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \right] \left\{ \begin{array}{c} -\frac{1}{4} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} \\ - \left[I - \left(\frac{1}{2} + \frac{1}{6} \right) \right] \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_1} \right)^{-1} \underline{g}^{E0} \, T_m \end{array} \right\} + \underline{g}^{E0}_{avg} \, T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + \frac{1}{4} C^{E0}_{Bvar_1} \left[\left(5 \, \Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_1} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_1} \right)^{-1} \underline{g}^{E0}_{avg} \, T_m$$

$$- \frac{1}{2} C^{E0}_{Bvar_1} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_1} \right)^{-1} \underline{g}^{E0}_{avg} \, T_m - \frac{1}{4} C^{E0}_{Bvar_1} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m}$$

$$+ C^{E0}_{Bvar_1} \left(\frac{1}{2} + \frac{1}{6} \right) \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \underline{g}^{E0}_{avg} \, T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_1} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times \right) \frac{\underline{l}^{Bref}}{T_m}$$

For $n > 1$:

$$\underline{V}^{E0}_{var_{2n-1}} = \underline{V}^{E0}_{var_{2n-2}} + C^{E0}_{Bvar_{2n-2}} G_{Vvar_{2n-1}} \Delta\underline{\upsilon}^{Bvar}_{var_{2n-1}} + \underline{g}^{E0}_{avg} T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{2n-3}} \left(\Delta\underline{\alpha}'^{Bvar}_{var} \times\right) \frac{\underline{l}^{Bref}}{T_m}$$

$$+ C^{E0}_{Bvar_{2n-2}} \left[I + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right] \left\{ \begin{array}{c} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \frac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\frac{1}{2} - \frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} \underline{g}^{E0} T_m \end{array} \right\} + \underline{g}^{E0}_{avg} T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{2n-2}} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_{2n-2}} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} \underline{g}^{E0}_{avg} T_m$$

(H-52)

$$\underline{V}^{E0}_{var_{2n}} = \underline{V}^{E0}_{var_{2n-1}} + C^{E0}_{Bvar_{2n-1}} G_{Vvar_{2n}} \Delta\underline{\upsilon}^{Bvar}_{var_{2n}} + \underline{g}^{E0}_{avg} T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{2n-2}} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_{2n-2}} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1} \underline{g}^{E0}_{avg} T_m$$

$$+ C^{E0}_{Bvar_{2n-1}} \left[I + \frac{1}{2}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right] \left\{ \begin{array}{c} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)^2 \frac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1} \underline{g}^{E0} T_m \end{array} \right\} + \underline{g}^{E0}_{avg} T_m$$

$$\approx \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{2n-1}} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1} \underline{g}^{E0}_{avg} T_m$$

$$- \frac{1}{2} C^{E0}_{Bvar_{2n-1}} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1} \underline{g}^{E0}_{avg} T_m$$

$$+ C^{E0}_{Bvar_{2n-1}} \left(\frac{1}{2} + \frac{1}{6}\right)\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1} \underline{g}^{E0}_{avg} T_m$$

$$= \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{2n-1}} \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right) \frac{\underline{l}^{Bref}}{T_m}$$

Summary of (H-47) to (H-52) Velocity Results:

$$\text{For } n < -4: \quad \underline{V}^{E0}_{var_{2n-1}} = \underline{V}^{E0}_{ref_0} \qquad \underline{V}^{E0}_{var_{2n}} = \underline{V}^{E0}_{ref_0}$$

$$\text{For } n = -4:$$

$$\underline{V}^{E0}_{var_{-9}} = \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_{-9}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{12} C^{E0}_{Bvar_{-9}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{-9}} \right)^{-1} \underline{g}^{E0}_{avg} T_m$$

$$\underline{V}^{E0}_{var_{-8}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{-9}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \frac{\underline{l}^{Bref}}{T_m}$$

$$\text{For } 0 > n > -4:$$

$$\underline{V}^{E0}_{var_{2n-1}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-2}} \right)^{-1} \underline{g}^{E0}_{avg} T_m$$

$$\underline{V}^{E0}_{var_{2n}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \frac{\underline{l}^{Bref}}{T_m}$$

$$\text{For } n = 0:$$

$$\underline{V}^{E0}_{var_{-1}} = \underline{V}^{E0}_{ref_0} + \frac{1}{4} C^{E0}_{Bvar_{-2}} \left[\left(5\, \Delta \underline{\alpha}^{Bvar}_{var} - \Delta \underline{\alpha}'^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m}$$

$$- \frac{1}{6} C^{E0}_{Bvar_{-2}} \left(\Delta \underline{\alpha}^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{-2}} \right)^{-1} \underline{g}^{E0}_{avg} T_m \qquad \text{(H-53)}$$

$$\underline{V}^{E0}_{var_0} = \underline{V}^{E0}_{ref_0} + \frac{1}{2} C^{E0}_{Bvar_{-1}} \left[\left(\Delta \underline{\alpha}'^{Bvar}_{var} + \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m}$$

$$\text{For } n = 1:$$

$$\underline{V}^{E0}_{var_1} = \underline{V}^{E0}_{ref_0} + \frac{1}{4} C^{E0}_{Bvar_0} \left[\left(5\, \Delta \underline{\alpha}'^{Bvar}_{var} - \Delta \underline{\alpha}^{Bvar}_{var} \right) \times \right] \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_0} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_0} \right)^{-1} \underline{g}^{E0}_{avg} T_m$$

$$\underline{V}^{E0}_{var_2} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_1} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \frac{\underline{l}^{Bref}}{T_m}$$

$$\text{For } n > 1:$$

$$\underline{V}^{E0}_{var_{2n-1}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-2}} \right)^{-1} \underline{g}^{E0}_{avg} T_m$$

$$\underline{V}^{E0}_{var_{2n}} = \underline{V}^{E0}_{ref_0} + C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \frac{\underline{l}^{Bref}}{T_m}$$

Recognizing that the inverse of a direction cosine matrix equals its transpose, the $\left(C^{E0}_{Bvar_n} \right)^{-1}$ terms in (H-53) become $C^{Bvar}_{E0_n}$, generating (50) in [1].

POSITION DETERMINATION

Approximating gravity for test example conditions as the constant $\underline{g}_{avg}^{E0}$, position from (4) with $m = 2n$ becomes at time instants m-1 and m:

$$\begin{aligned} \underline{R}_{var_{2n-1}}^{E0} &= \underline{R}_{var_{2n-2}}^{E0} + \underline{V}_{var_{2n-2}}^{E0} T_m + C_{Bvar_{2n-2}}^{E0} G_{Rvar_{2n-1}} \Delta\underline{\upsilon}_{var_{2n-1}}^{Bvar} T_m + \frac{1}{2} \underline{g}_{avg}^{E0} T_m^2 \\ \underline{R}_{var_{2n}}^{E0} &= \underline{R}_{var_{2n-1}}^{E0} + \underline{V}_{var_{2n-1}}^{E0} T_m + C_{Bvar_{2n-1}}^{E0} G_{Rvar_{2n}} \Delta\underline{\upsilon}_{var_{2n}}^{Bvar} T_m + \frac{1}{2} \underline{g}_{avg}^{E0} T_m^2 \end{aligned} \quad \text{(H-54)}$$

Initializing $\underline{R}_{var_{2n-1}}^{E0}$ position (i.e., $\underline{R}_{var_{2n-2}}^{E0}$) for $n < -4$ derives directly from (37) with $m = 2n-2$, and $C_{Bvar_m}^{E0} = C_{Bvar_{-9}}^{E0}$ from (32):

$$\text{For } n < -4: \quad \underline{R}_{var_{2n-2}}^{E0} = (2n-2)\, \underline{V}_{ref_0}^{E0} T_m + C_{Bvar_{-9}}^{E0} l^{Bref} \qquad \text{(H-55)}$$

With $\underline{R}_{var_{2n-2}}^{E0}$ from (H-55) for $n < -4$, (H-53) for $\underline{V}_{var}^{E0}$ velocity, (H-45) for $\Delta\underline{\upsilon}_{var}^{Bvar}$ specific force, (32) for C_{Bvar}^{E0} attitude, and (33) for G_{Rvar}, (H-54) becomes to first order accuracy for position:

$$\text{For } n < -4:$$

$$\begin{aligned} \underline{R}_{var_{2n-1}}^{E0} &= \underline{R}_{var_{2n-2}}^{E0} + \underline{V}_{var_{2n-2}}^{E0} T_m + C_{Bvar_{2n-2}}^{E0} G_{Rvar_{2n-1}} \Delta\underline{\upsilon}_{var_{2n-1}}^{Bvar} T_m + \frac{1}{2} \underline{g}_{avg}^{E0} T_m^2 \\ &= (2n-2)\, \underline{V}_{ref_0}^{E0} T_m + C_{Bvar_{-9}}^{E0} l^{Bref} + \underline{V}_{ref_0}^{E0} T_m - C_{Bvar_{2n-2}}^{E0} \frac{1}{2} \left(C_{Bvar_{-9}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m^2 + \frac{1}{2} \underline{g}_{avg}^{E0} T_m^2 \\ &= (2n-1)\, \underline{V}_{ref_0}^{E0} T_m + C_{Bvar_{-9}}^{E0} l^{Bref} \\ \underline{R}_{var_{2n}}^{E0} &= \underline{R}_{var_{2n-1}}^{E0} + \underline{V}_{var_{2n-1}}^{E0} T_m + C_{Bvar_{2n-1}}^{E0} G_{Rvar_{2n}} \Delta\underline{\upsilon}_{var_{2n}}^{Bvar} T_m + \frac{1}{2} \underline{g}_{avg}^{E0} T_m^2 \\ &= (2n-1)\, \underline{V}_{ref_0}^{E0} T_m + C_{Bvar_{-9}}^{E0} l^{Bref} + \underline{V}_{ref_0}^{E0} T_m - C_{Bvar_{-9}}^{E0} \frac{1}{2} \left(C_{Bvar_{-9}}^{E0} \right)^{-1} \underline{g}_{avg}^{E0} T_m T_m + \frac{1}{2} \underline{g}_{avg}^{E0} T_m^2 \\ &= 2n\, \underline{V}_{ref_0}^{E0} T_m + C_{Bvar_{-9}}^{E0} l^{Bref} \end{aligned} \quad \text{(H-56)}$$

For $n = -4$:

$$\underline{R}^{E0}_{var-9} = \underline{R}^{E0}_{var-10} + \underline{V}^{E0}_{var-10} T_m + C^{E0}_{Bvar-10} G_{Rvar-9} \Delta\underline{\upsilon}^{Bvar}_{var-9} T_m + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2$$

$$= -10\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar-9} l^{Bref} + \underline{V}^{E0}_{ref_0} T_m$$

$$+ C^{E0}_{Bvar-9} \frac{1}{2} \left\{ \begin{array}{c} \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right) \frac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\frac{1}{4} - \frac{1}{3}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar-9}\right)^{-1} \underline{g}^{E0} T_m \end{array} \right\} T_m + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2 \qquad \text{(H-57)}$$

$$= -9\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar-9}\left[I + \frac{1}{4}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right] \underline{l}^{Bref}$$

$$+ \frac{1}{2}\left(\frac{1}{4} - \frac{1}{3}\right) C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1} \underline{g}^{E0}_{avg} T_m^2$$

(Continued)

(H-57) Concluded

$$
\begin{aligned}
\underline{R}^{E0}_{var-8} &= \underline{R}^{E0}_{var-9} + \underline{V}^{E0}_{var-9}\, T_m + C^{E0}_{Bvar-9}\, G_{Rvar-8}\, \Delta\underline{\upsilon}^{Bvar}_{var-8}\, T_m + \frac{1}{2}\underline{g}^{E0}_{avg}\, T_m^2 \\
&= -9\,\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-9}\left[I + \frac{1}{4}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\underline{l}^{Bref} + \underline{V}^{E0}_{ref_0}\, T_m \\
&\quad + \frac{1}{2}\left(\frac{1}{4} - \frac{1}{3}\right) C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2 \\
&\quad + \frac{1}{2}\, C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} - \frac{1}{12} C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2 \\
&\quad + C^{E0}_{Bvar-9}\,\frac{1}{2}\left[I + \frac{1}{3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left\{\begin{array}{c} \frac{1}{2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\dfrac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\frac{1}{4} + \frac{1}{3}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}\, T_m \end{array}\right\} T_m + \frac{1}{2}\underline{g}^{E0}_{avg}\, T_m^2 \\
&= -8\,\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-9}\left[I + \frac{1}{4}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\underline{l}^{Bref} + \frac{1}{2}\left(\frac{1}{4} - \frac{1}{3}\right) C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2 \\
&\quad + \frac{1}{2}\, C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} - \frac{1}{12} C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2 \\
&\quad - C^{E0}_{Bvar-9}\,\frac{1}{6}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}_{avg} \\
&\quad + \frac{1}{4} C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} + \frac{1}{2}\left(\frac{1}{4} + \frac{1}{3}\right) C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2 \\
&= -8\,\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-9}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\underline{l}^{Bref} + \frac{1}{2}\left(\frac{1}{4} - \frac{1}{3}\right) C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}\, T_m^2 \\
&\quad - \frac{1}{12} C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2 - \frac{1}{6} C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}_{avg} \\
&\quad + \frac{1}{2}\left(\frac{1}{4} + \frac{1}{3}\right) C^{E0}_{Bvar-9}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-9}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2 \\
&= -8\,\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-9}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\underline{l}^{Bref} \\
&= -8\,\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-8}\,\underline{l}^{Bref}
\end{aligned}
$$

For $0 > n > -4$:

$$\underline{R}^{E0}_{var_{2n-1}} = \underline{R}^{E0}_{var_{2n-2}} + \underline{V}^{E0}_{var_{2n-2}} T_m + C^{E0}_{Bvar_{2n-2}} G_{Rvar_{2n-1}} \Delta\underline{\upsilon}^{Bvar}_{var_{2n-1}} T_m + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2$$

$$= (2n-2)\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-2}} \underline{l}^{Bref} + \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-3}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$

$$+ C^{E0}_{Bvar_{2n-2}} \frac{1}{2}\left[I + \frac{1}{3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left\{\begin{array}{c}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \dfrac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\dfrac{1}{2}-\dfrac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0} T_m\end{array}\right\} T_m$$

$$+ \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2 \qquad \text{(H-58)}$$

$$\approx (2n-1)\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-2}} \underline{l}^{Bref} + C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$

$$-\frac{1}{6} C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg} T_m^2$$

$$+\frac{1}{2}\left(\frac{1}{2}-\frac{1}{6}\right) C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg} T_m^2$$

$$= (2n-1)\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-2}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\underline{l}^{Bref}$$

$$= (2n-1)\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}} \underline{l}^{Bref}$$

(Continued)

(H-58) Concluded

$$
\begin{aligned}
\underline{R}^{E0}_{var_{2n}} &= \underline{R}^{E0}_{var_{2n-1}} + \underline{V}^{E0}_{var_{2n-1}} T_m + C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} \Delta\underline{\upsilon}^{Bvar}_{var_{2n}} T_m + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2 \\
&= (2n-1)\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}} \underline{l}^{Bref} \\
&\quad + \underline{V}^{E0}_{ref_0} T_m + \left\{ \begin{array}{c} C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\dfrac{\underline{l}^{Bref}}{T_m} \\ -\dfrac{1}{6} C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg} T_m \end{array} \right\} T_m \\
&\quad + C^{E0}_{Bvar_{2n-1}} \frac{1}{2}\left[I + \frac{1}{3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right] \left\{ \begin{array}{c} \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)^2 \dfrac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\dfrac{1}{2}+\dfrac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\underline{g}^{E0}_{avg} T_m \end{array} \right\} T_m + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2 \\
&\approx 2n\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}} \underline{l}^{Bref} + C^{E0}_{Bvar_{2n-1}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} \\
&\quad -\frac{1}{6} C^{E0}_{Bvar_{2n-2}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg} T_m^2 \\
&\quad -\frac{1}{6} C^{E0}_{Bvar_{2n-1}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\underline{g}^{E0}_{avg} T_m^2 \\
&\quad + \frac{1}{2}\left(\frac{1}{2}+\frac{1}{6}\right) C^{E0}_{Bvar_{2n-1}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-1}}\right)^{-1}\underline{g}^{E0}_{avg} T_m^2 + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2 \\
&= 2n\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}}\left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\underline{l}^{Bref} \\
&= 2n\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n}} \underline{l}^{Bref}
\end{aligned}
$$

For $n = 0$:

$$\underline{R}^{E0}_{var-1} = \underline{R}^{E0}_{var-2} + \underline{V}^{E0}_{var-2}\, T_m + C^{E0}_{Bvar-2}\, G_{Rvar-1}\, \Delta\underline{\upsilon}^{Bvar}_{var-1}\, T_m + \frac{1}{2}\underline{g}^{E0}_{avg}\, T_m^2$$

$$= -2\,\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-2}\, \underline{l}^{Bref} + \underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\frac{\underline{l}^{Bref}}{T_m}\, T_m$$

$$+ C^{E0}_{Bvar-2}\,\frac{1}{2}\left[I + \frac{1}{3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left\{\begin{array}{c} -\frac{1}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\frac{1}{2} - \frac{1}{6}\right)\right]\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m \end{array}\right\} T_m$$

$$+\frac{1}{2}\underline{g}^{E0}_{avg}\, T_m^2 \qquad \text{(H-59)}$$

$$\approx -\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-2}\, \underline{l}^{Bref} + C^{E0}_{Bvar-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref}$$

$$-\frac{1}{6}C^{E0}_{Bvar-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2$$

$$-\frac{1}{8}C^{E0}_{Bvar-2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} + C^{E0}_{Bvar-2}\left(\frac{1}{4} - \frac{1}{12}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar-2}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2$$

$$= -\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-2}\, \underline{l}^{Bref} + C^{E0}_{Bvar-2}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\underline{l}^{Bref} - \frac{1}{8}C^{E0}_{Bvar-2}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref}$$

$$\approx -\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-1}\, \underline{l}^{Bref} - \frac{1}{8}C^{E0}_{Bvar-1}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref}$$

$$= -\underline{V}^{E0}_{ref_0}\, T_m + C^{E0}_{Bvar-1}\left\{I - \frac{1}{8}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{l}^{Bref}$$

(Continued)

(H-59) Concluded

$$\underline{R}^{E0}_{var_0} = \underline{R}^{E0}_{var_{-1}} + \underline{V}^{E0}_{var_{-1}} T_m + C^{E0}_{Bvar_{-1}} G_{Rvar_0} \Delta\underline{\upsilon}^{Bvar}_{var_0} T_m + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2$$

$$= -\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{-1}} \left\{ I - \frac{1}{8}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\} \underline{l}^{Bref}$$

$$+ \underline{V}^{E0}_{ref_0} T_m + \left\{ \frac{1}{4} C^{E0}_{Bvar_{-2}} \left[\left(5\,\Delta\underline{\alpha}^{Bvar}_{var} - \Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right] \frac{\underline{l}^{Bref}}{T_m} - \frac{1}{6} C^{E0}_{Bvar_{-2}} \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-2}}\right)^{-1} \underline{g}^{E0}_{avg} T_m \right\}$$

$$+ C^{E0}_{Bvar_{-1}} \frac{1}{2}\left[I + \frac{1}{3}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right] \left\{ \begin{array}{c} \frac{3}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\frac{\underline{l}^{Bref}}{T_m} \\ -\left[I - \left(\frac{1}{2}+\frac{1}{6}\right)\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\left(C^{E0}_{Bvar_{-1}}\right)^{-1} \underline{g}^{E0} T_m \end{array} \right\} T_m + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2$$

$$\approx C^{E0}_{Bvar_{-1}} \left\{ I - \frac{1}{8}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\} \underline{l}^{Bref}$$

$$+ \frac{1}{4} C^{E0}_{Bvar_{-1}} \left[\left(5\,\Delta\underline{\alpha}^{Bvar}_{var} - \Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right] \underline{l}^{Bref} - \frac{1}{6} C^{E0}_{Bvar_{-1}} \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-1}}\right)^{-1} \underline{g}^{E0}_{avg} T_m$$

$$- \frac{1}{6} C^{E0}_{Bvar_{-1}} \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-1}}\right)^{-1} \underline{g}^{E0} T_m + \frac{3}{8} C^{E0}_{Bvar_{-1}} \left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \underline{l}^{Bref}$$

$$+ \frac{1}{2}\left(\frac{1}{2}+\frac{1}{6}\right) C^{E0}_{Bvar_{-1}} \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-1}}\right)^{-1} \underline{g}^{E0}_{avg} T_m^2$$

$$= C^{E0}_{Bvar_{-1}} \left[I + \left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right] \underline{l}^{Bref}$$

$$= C^{E0}_{Bvar_0} \underline{l}^{Bref}$$

For $n = 1$:

$$\underline{R}^{E0}_{var_1} = \underline{R}^{E0}_{var_0} + \underline{V}^{E0}_{var_0} T_m + C^{E0}_{Bvar_0} G_{Rvar_1} \Delta\underline{\upsilon}^{Bvar}_{var_1} T_m + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2$$

$$= \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_0}\underline{l}^{Bref} + \frac{1}{2}C^{E0}_{Bvar_{-1}}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref}$$

$$+ C^{E0}_{Bvar_0}\frac{1}{2}\left[I + \frac{1}{3}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\left\{\begin{array}{c}\frac{3}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} \\ -\left[I - \left(\frac{1}{2} - \frac{1}{6}\right)\right]\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_0}\right)^{-1}\underline{g}^{E0} T_m^2\end{array}\right\} + \frac{1}{2}\underline{g}^{E0}_{avg} T_m^2$$

$$\approx \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_0}\underline{l}^{Bref} + \frac{1}{2}C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} + \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} \qquad \text{(H-60)}$$

$$-\frac{1}{6}C^{E0}_{Bvar_0}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_0}\right)^{-1}\underline{g}^{E0}_{avg} T_m^2$$

$$+ \frac{3}{8}C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref} + \frac{1}{2}\left(\frac{1}{2} - \frac{1}{6}\right)C^{E0}_{Bvar_0}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{2n-2}}\right)^{-1}\underline{g}^{E0}_{avg} T_m^2$$

$$= \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_0}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right]\underline{l}^{Bref} - \frac{1}{8}C^{E0}_{Bvar_0}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\underline{l}^{Bref}$$

$$\approx \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_1}\left\{I - \frac{1}{8}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{l}^{Bref}$$

(Continued)

(H-60) Concluded

$$\underline{R}^{E0}_{var_2} = \underline{R}^{E0}_{var_1} + \underline{V}^{E0}_{var_1} T_m + C^{E0}_{Bvar_1} G_{Rvar_2} \Delta\underline{\upsilon}^{Bvar}_{var_2} T_m + \frac{1}{2} \underline{g}^{E0}_{avg} T_m^2$$

$$= \underline{V}^{E_0}_{ref_0} T_m + C^{E0}_{Bvar_1} \left\{ I - \frac{1}{8}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\} \underline{l}^{Bref} + \underline{V}^{E_0}_{ref_0} T_m$$

$$+ \left\{ \begin{array}{l} \frac{1}{4} C^{E0}_{Bvar_0}\left[\left(5\,\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \underline{l}^{Bref} \\ -\frac{1}{6} C^{E0}_{Bvar_0}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_0}\right)^{-1} \underline{g}^{E0} T_m^2 \end{array} \right\}$$

$$+ C^{E0}_{Bvar_1} \frac{1}{2}\left[I + \frac{1}{3}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\right] \left\{ \begin{array}{c} -\frac{1}{4}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \underline{l}^{Bref} \\ -\left[I - \left(\frac{1}{2} + \frac{1}{6}\right)\right]\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_1}\right)^{-1} \underline{g}^{E0}_{avg} T_m^2 \end{array} \right\} + \frac{1}{2} \underline{g}^{E0}_{avg} T_m^2$$

$$\approx 2\, \underline{V}^{E_0}_{ref_0} T_m + C^{E0}_{Bvar_1} \left\{ I - \frac{1}{8}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\} \underline{l}^{Bref} + \frac{1}{4} C^{E0}_{Bvar_1}\left[\left(5\,\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \underline{l}^{Bref}$$

$$-\frac{1}{6} C^{E0}_{Bvar_1}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_1}\right)^{-1} \underline{g}^{E0}_{avg} T_m^2 - \frac{1}{6} C^{E0}_{Bvar_1}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_1}\right)^{-1} \underline{g}^{E0}_{avg} T_m^2$$

$$-\frac{1}{8} C^{E0}_{Bvar_1}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right] \underline{l}^{Bref}$$

$$+\frac{1}{2}\left(\frac{1}{2} + \frac{1}{6}\right) C^{E0}_{Bvar_1}\left(\Delta\underline{\alpha}'^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_1}\right)^{-1} \underline{g}^{E0}_{avg} T_m^2$$

$$= 2\, \underline{V}^{E_0}_{ref_0} T_m + C^{E0}_{Bvar_1}\left[I + \left(\Delta\underline{\alpha}'^{Bvar}_{var}\right)\times\right] \underline{l}^{Bref}$$

$$= 2\, \underline{V}^{E_0}_{ref_0} T_m + C^{E0}_{Bvar_2} \underline{l}^{Bref}$$

For $n > 1$:

$$\underline{R}^{E0}_{var_{2n-1}} = \underline{R}^{E0}_{var_{2n-2}} + \underline{V}^{E0}_{var_{2n-2}} T_m + C^{E0}_{Bvar_{2n-2}} G_{Rvar_{2n-1}} \Delta \underline{\upsilon}^{Bvar}_{var_{2n-1}} T_m + \frac{1}{2} \underline{g}^{E0}_{avg} T_m^2$$
$$= (2n-2) \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-2}} \underline{l}^{Bref} + \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{n-3}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \underline{l}^{Bref}$$
$$+ C^{E0}_{Bvar_{2n-2}} \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] \left\{ \begin{array}{c} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^2 \underline{l}^{Bref} \\ - \left[I - \left(\frac{1}{2} - \frac{1}{6} \right) \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] \left(C^{E0}_{Bvar_{2n-2}} \right)^{-1} \underline{g}^{E0} T_m^2 \end{array} \right\} + \frac{1}{2} \underline{g}^{E0}_{avg} T_m^2$$
$$\approx (2n-1) \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-2}} \underline{l}^{Bref} + C^{E0}_{Bvar_{n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \underline{l}^{Bref}$$
$$= (2n-1) \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}} \underline{l}^{Bref}$$

(H-61)

$$\underline{R}^{E0}_{var_{2n}} = \underline{R}^{E0}_{var_{2n1}} + \underline{V}^{E0}_{var_{2n-1}} T_m + C^{E0}_{Bvar_{2n-1}} G_{Rvar_{2n}} \Delta \underline{\upsilon}^{Bvar}_{var_{2n}} T_m + \frac{1}{2} \underline{g}^{E0}_{avg} T_m^2$$
$$= (2n-1) \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}} \underline{l}^{Bref} + \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \underline{l}^{Bref}$$
$$- \frac{1}{6} C^{E0}_{Bvar_{2n-2}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-2}} \right)^{-1} \underline{g}^{E0}_{avg} T_m^2$$
$$+ C^{E0}_{Bvar_{2n-1}} \frac{1}{2} \left[I + \frac{1}{3} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] \left\{ \begin{array}{c} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right)^2 \underline{l}^{Bref} \\ - \left[I - \left(\frac{1}{2} + \frac{1}{6} \right) \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \right] \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \underline{g}^{E0}_{avg} T_m^2 \end{array} \right\} + \frac{1}{2} \underline{g}^{E0}_{avg} T_m^2$$
$$\approx 2n \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}} \underline{l}^{Bref} + C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \underline{l}^{Bref}$$
$$- \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \underline{g}^{E0}_{avg} T_m^2$$
$$- \frac{1}{6} C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \underline{g}^{E0}_{avg} T_m^2$$
$$+ C^{E0}_{Bvar_{2n-1}} \frac{1}{2} \left(\frac{1}{2} + \frac{1}{6} \right) \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \left(C^{E0}_{Bvar_{2n-1}} \right)^{-1} \underline{g}^{E0}_{avg} T_m^2$$
$$= 2n \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}} \underline{l}^{Bref} + C^{E0}_{Bvar_{2n-1}} \left(\Delta \underline{\alpha}'^{Bvar}_{var} \times \right) \underline{l}^{Bref}$$
$$= 2n \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n}} \underline{l}^{Bref}$$

Summary of (H-56) of (H-61) Position Results:

For $n < -4$:

$$\underline{R}^{E0}_{var_{2n-1}} = (2n-1)\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{-9}} \underline{l}^{Bref} \qquad \underline{R}^{E0}_{var_{2n}} = 2n\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{-9}} \underline{l}^{Bref}$$

For $n = -4$:

$$\underline{R}^{E0}_{var_{-9}} = -9\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{-9}}\left[I + \frac{1}{4}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\right]\underline{l}^{Bref}$$

$$+\frac{1}{2}\left(\frac{1}{4}-\frac{1}{3}\right) C^{E0}_{Bvar_{-9}}\left(\Delta\underline{\alpha}^{Bvar}_{var}\times\right)\left(C^{E0}_{Bvar_{-9}}\right)^{-1}\underline{g}^{E0}_{avg}\, T_m^2$$

$$\underline{R}^{E0}_{var_{-8}} = -8\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{-8}} \underline{l}^{Bref}$$

For $0 > n > -4$:

$$\underline{R}^{E0}_{var_{2n-1}} = (2n-1)\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}} \underline{l}^{Bref} \qquad \text{(H-62)}$$

$$\underline{R}^{E0}_{var_{2n}} = 2n\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n}} \underline{l}^{Bref}$$

For $n = 0$:

$$\underline{R}^{E0}_{var_{-1}} = -\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{-1}}\left\{ I - \frac{1}{8}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{l}^{Bref} \qquad \underline{R}^{E0}_{var_0} = C^{E0}_{Bvar_0} \underline{l}^{Bref}$$

For $n = 1$:

$$\underline{R}^{E0}_{var_1} = \underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_1}\left\{ I - \frac{1}{8}\left[\left(\Delta\underline{\alpha}'^{Bvar}_{var} - \Delta\underline{\alpha}^{Bvar}_{var}\right)\times\right]\right\}\underline{l}^{Bref}$$

$$\underline{R}^{E0}_{var_2} = 2\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_2} \underline{l}^{Bref}$$

For $n > 1$:

$$\underline{R}^{E0}_{var_{2n-1}} = (2n-1)\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n-1}} \underline{l}^{Bref} \qquad \underline{R}^{E0}_{var_{2n}} = 2n\,\underline{V}^{E0}_{ref_0} T_m + C^{E0}_{Bvar_{2n}} \underline{l}^{Bref}$$

Recognizing that the inverse of a direction cosine matrix equals its transpose, the $\left(C^{E0}_{Bvar_n}\right)^{-1}$ terms in (H-62) become $C^{Bvar}_{E0_n}$, generating (51) in [1].

REFERENCES

[1] Savage, Paul G., "Generating Strapdown Specific-Force/Angular-Rate For Specified Attitude/ Position Variation From A Reference Trajectory", SAI WBN-14026, www.strapdownassociates.com, April 21, 2020.
http://www.strapdownassociates.com/Variation%20Trajectory%20Generator.pdf